Economic Policy and Equality of Opportunity *

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Abstract

We employ equality of opportunity (EOP) definitions from the literature on distributive justice to a quantitative model featuring intergenerational human capital investments and luck. When calibrated to the U.S., the model-implied degree of EOP differs substantially depending on whether one consider it ethical to reward offspring for the effort of previous generations. Despite reducing intragenerational inequality, education subsidies do little to promote EOP. This is because if one thinks intergenerational investments should be rewarded, there is little room for improvement to begin with; In the opposite case, much stronger redistribution is needed for the policies to have a quantitative impact.

We present a quantitative economic model of human capital investment within and across generations, with incomplete markets and government transfer programs. We then map the outcome distribution of the model into existing notions of equality that have appeared in the philosophical literature on distributive justice. Our analysis is novel in that we provide an operationalisation of philosophical concepts in a quantitative economic framework. The advantage of our model-based approach is that we can apply these notions to

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objects that are discussed in the abstract but for which no such data exists (unobservable genetic traits, effects from and to far-away generations). A counterfactual policy analysis shows that egalitarian policies that target the equality of outcomes have little impact on equality of opportunity (EOP).

(In)equality is at the heart of many branches of the social sciences. A major challenge in philosophy is to establish an ethically defensible notion of equality that a society should strive for. There is an implicit consensus that outcomes should not depend on circumstances that are beyond an individual's control, but individual efforts given such circumstances should be rewarded (Arneson, 1989; Roemer, 1993). Put simply, opportunities should be equalized while responsibility should be rewarded. From this it also follows that equal efforts should be rewarded equally. This hardly solves the problem, since we then need to be able to identify what constitutes an opportunity that an individual is not responsible for, and have a notion of how to compare efforts from individuals with different circumstances. Even equipped with such definitions, obstacles abound when trying to implement such notions in practice. Lefranc *et al.* (2008, 2009) are some of the few empirical attempts, who operationalise successive developments since Roemer.

Mainstream economics has taken a different route. Despite some early discussions among economists (Atkinson, 1970; Sen, 1970; Okun, 1975), the main emphasis in neoclassical theory was on economic efficiency rather than equality. This is well manifested in an influential paper by Lucas (2003). Existing normative studies on economic inequality and redistribution either rely on Pareto improvements or assume a social welfare function. The latter is typically a Benthamian-type average utility which is maximized by a social planner. In this case, the socially optimal outcome becomes a mixture of maximizing aggregate efficiency while minimizing disparate jumps in the individuals' consumption paths over time.

The emphasis on efficiency notwithstanding, economists have long been interested in measures of inequality such as Gini coefficients, Lorenz curves, and percentile income concentrations, and analysed how these objects would change in response to different policies. Recent research puts more emphasis on positive than normative analysis, i.e., to compute the costs and benefits of potential policies at the disposal of a social planner given her objective function, not what the planner's objective should be. And quantitative economic research is able to incorporate degrees of heterogeneity that were previously unimaginable. However, to the best of our knowledge, no attempts have been made to apply egalitarian notions of EOP in quantitative research, which still mainly relies on utilitarian welfare criteria.

Our contribution is two-fold: for philosophers, we present a model-based approach which can serve as a test-bed for EOP notions that can be difficult to recover directly from the data. For economists, we illustrate that the "veil of ignorance" criterion far from reflects egalitarian concerns, as has been critiqued by Roemer (2002). We *do not* develop new notions of EOP, but rather define, for a given generation in the model, which variables should be included as circumstantial (for which an individual should not be rewarded; also referred to as "types") vis a vis outcomes. We also show that reducing inequality of outcomes is not equivalent to increasing EOP.

Clearly, we are not the first to apply philosophical concepts to economics, but to the best of our knowledge, all previous attempts have been purely empirical. Lefranc *et al.* (2008) posit a philosophical definition of EOP owing to Roemer (1993) and extend it to an empirically testable definition which incorporates "luck," which is also important for our analysis. Applying this to French data in 1979-2000, they conclude that there is a significant degree of inequality in observables that can be categorized as circumstantial, while "luck" seems to be rewarded equally conditional on circumstances. Since one cannot hope to be able to equalize all returns to luck, such an outcome is ethically desirable.

We proceed in the opposite direction. After presenting a quantitative economic model calibrated to the 1990 U.S., we explain how we would measure the degree of EOP from the model, employing existing definitions. The model incorporates multiple stages of human capital investment during childhood, a college decision, and on-the-job investments in adulthood. This emphasis on human capital acquisition is in line with recent studies that point to human capital as a major source of inequality (Huggett *et al.*, 2011), which is especially sensitive to investments at earlier ages (Cunha *et al.*, 2010; Corak, 2013).[1] In contrast to empirical studies in which luck is the residual outcome not explained by observables (in an OLS sense), our model features two sources of fundamental luck: genetic luck and market luck. The effect of such luck cannot be directly measured in the data, as it affects individual decisions that translate non-linearly into (market) outcomes.

Using the recursive representation of most quantitative macroeconomic models, circumstantial variables can be mapped into the states of the value function, or previous choices that led to those states. In our model, the education, human capital, and wealth of the parent correspond to "types" that can be observed in the data, while the child's (lifetime) earnings and wealth correspond to outcomes. EOP is measured by comparing the conditional distribution of children's outcomes by type. With full EOP, these distributions should overlap.

Despite advances in quantitative research, it is difficult to incorporate all abstract notions of "circumstances" or "effort" into a single model.[2] We are not an exception, and we assume that all such objects are assumed to be the same across all individuals in the economy.[3] Instead we emphasize that our model, which builds on standard economic theories of education and economic outcomes, addresses at least two concerns that are hard to implement in empirical studies in practice:

[1]The benchmark model also includes education and lump-sum subsidies financed by a progressive income tax, and a PAYGO social security system.

[2]For example, most economic models allow little heterogeneity in preferences, technology, and/or belief formation, which are important to distinguish in the distributive justice literature (Cohen, 1989).

[3]Assuming all individuals share the same belief about the state of the world is not as heroic as it may sound, at least for long-run outcomes, which we focus on. In Section 3.5, we discuss the implications of adding preference heterogeneity in the context of our model.

1. how to compare inequality across generations, and
2. differentiate ex ante determined ability vis a vis ex post accumulated human capital.

This is not to say that we overcome the empirical difficulties faced in the literature, but rather, we shed light on what theoretical predictions can be tested if the relevant data becomes available.

In particular, we focus on a question which is central to economics and philosophy (Arneson, 1998): if a well-educated parent affords better education for her child, should this be viewed as better circumstances for the child, which should be equalized away (an *individualistic view*), or the effort of the grandparent, which should be rewarded (a *dynastic view*)? When analysing egalitarian policies, it is important to focus not only on how the distribution of parental circumstances affect the next generation, but also how parental *efforts* affect the next generation.

While we do not give an answer to this question, we argue that different type-outcome pairs should be considered when measuring EOP depending on which stance is taken. Specifically, if one takes the dynastic view, we should look at children's lifetime earnings conditional on parental luck or ability. If one takes the individualistic view, we should look at children's net wealth when the children are of an age in which all investments in the *grandchildren* have been completed, conditional on parents' lifetime earnings. We corroborate our argument by showing that such choice of pairs correctly reflect the distribution of continuation utilities, which is available in our structural model but unobserved in the data.[4]

Following existing notions of EOP, we first analyse the conditional distributions of the next generation's lifetime earnings by the lifetime earnings of the parent. But under a dynas-

[4] So in some sense, we are claiming what ought to be equalized (an *equalisandum*) according to an egalitarian stance (Cohen, 1989). From the dynastic viewpoint, parental efforts should be rewarded, while they should not be from an individual viewpoint. We thank an anonymous referee for pointing this out.

tic view, such variables themselves are at least partially the outcome of efforts from *previous* generations, that is, the grandparents, great-grandparents, etc. (Swift, 2004; Jusot *et al.*, 2013). So we extract outcome distributions that are conditional on a purely circumstantial variable in the model: the luck of the parents. The results are striking: conditional on parental luck as opposed to their lifetime earnings, EOP is almost achieved, in the sense that the conditional distributions converge toward each other. The culprit is our multi-layered human capital acquisition technology—luck is overwhelmed by intergenerational human capital investments.

Under an individualistic view, it is unethical to reward a future generation for accumulated efforts from distant ancestors. But if we ignore the fact that circumstantial variables at least partially reflect the effort of previous generations, the current generation should not be punished for what previous generations were not rewarded for (investment in children). We argue that the appropriate outcome variable we should look at, then, is not children's earnings but the wealth level of the child at a later age, net of transfers to his own offspring. Surprisingly, we find that despite net wealth being more equal than lifetime earnings within any given generation, EOP is smaller in the sense that their distributions conditional on parents' lifetime earnings become farther from each other. This is mainly due to the lowest type: because of the intergenerational borrowing constraint (non-negative bequest constraint), poor households persistently want to borrow from their children but cannot. This is also manifested as a wider variation in the conditional distributions of lower-types.

Individuals in our model are fully rational. Consequently, parents invest in children because they derive utility from doing so. So regardless of whether one views circumstantial variables as the outcome of efforts from previous generations or not, conditional *continuation utilities* are the appropriate outcomes to be compared, since it takes into account both the costs and benefits of having richer offspring.[5] The distributions of continuation utilities

[5]This has less to do with the expected utility notions emphasized by Vallentyne (2002) and others, and is

conditional on either parents' abilities or lifetime earnings closely mirror the two cases in the preceding paragraphs. Thus, it theoretically confirms our emphasis that intergenerational investments should be accounted for when comparing measures of EOP. Empirically, it calls for the need to estimate either genetic abilities or the net wealth of individuals in order for EOP to reflect utilities rather than dollar outcomes.

Finally, we analyse how EOP would change in response to a change in a government policy by contrasting the benchmark outcome distribution to the unobserved, structural distribution. Our model-based approach permits a joint analysis of different measures of EOP and utilitarian welfare, and the equilibrium approach enables us to focus on long-run outcomes, which are typically the goal of policies meant to promote EOP over generations. Hence the policy exercises can guide a policy-maker who faces both efficiency and egalitarian concerns in the long-run. To weigh such concerns, a cardinal measure is needed, and we advocate the usage of the Theil index as a measure of "IOP," or inequality of opportunity. The Theil index maps nicely into both philosophical and economic concepts. First, because it is readily decomposable, is is suitable for measuring compensation between and within types, which has been the major platform of redistributive justice debates. Second, it can be used as an input into the social welfare function of a planner who is concerned with both equity and efficiency in which the planner's weight on equity can be represented by an exogenous parameter.[6]

Since we keep the model and parameter values constant, we cannot deconstruct how much of inequality stems from the transmission technology (human capital acquisition) vis a vis the distribution of parental types. The advantage, however, is that the framework serves as a testbed for "controlled experiments": we can attribute all the shifts in the parents'

more about dynamics: The point is that the continuation value of subsequent generations must be taken into account, not the expected utilities at a single point in time.

[6]We also compare the index with conditional log variances, which is more commonly used in the literature (Ferreira and Gignoux, 2011).

distribution and resulting measures of EOP to the exogenously changing policy variable. Furthermore, we can analyse how the transmission technology is indirectly affected by the changing distribution of types (through policy and equilibrium effects), by comparing short- and long-run outcomes of a policy change.

The results are as follows: while education subsidies have large, positive effects on utilitarian welfare and equality of outcomes, it does little to promote EOP. This is for two reasons: when one takes the view that intergenerational efforts should be rewarded, there is little room for improvement in EOP to begin with. On the other hand when one takes the view that intergenerational efforts should not be rewarded, much stronger redistribution is needed for the policy to have a quantitative impact. For both policies, the short-run effects are negligible compared to the long-run effects, indicating policy changes targeting next generation outcomes can take a long time to manifest.

Our results depend on the values of the deep parameters of the model, and even more on the posited model itself. But given that the individual elements of our model are widely accepted and that the model economy closely replicates the data, our message should not be ignored. But even if one rejects our model, it should be clear that simple observables may not properly measure abstract notions of EOP, not only because of the lack of data but also because the true data-generating process is non-linear. The main message is that depending on one's notion of which parental circumstances ought to be compensated for and how intergenerational efforts are controlled for, the level of EOP can vary vastly (Fleurbaey, 1995); and that at least in the U.S., much stronger redistribution is called for to achieve EOP if one is to compensate for intergenerational investments (or the lack thereof). A utilitarian criterion may not sufficiently capture such egalitarian concerns (Arneson, 1999; Roemer, 2002).

The rest of the paper is organized as follows. Sections 1-2 lays out our model and numerical calibration. In Section 3, we spell out how we apply existing EOP notions to our model.

Section 4 presents the outcome of the application and our counterfactual policy analysis. Section 5 concludes.

1. Model

The model is a simplified version of Lee and Seshadri (2016), modified to facilitate the mapping of variables into EOP notions of interest.[7] We combine a standard life-cycle model of human capital accumulation à la Ben-Porath (1967) with an intergenerational transmission mechanism à la Becker and Tomes (1986). Individuals differ in their genetic ability and human capital, but they share identical preferences and face the same technologies. Ability is transmitted across generations, and pre-labour market initial conditions are determined by endogenous parental investments. Bequests occur after the parent completes child investments. Parents' anticipation of the life-cycle decisions their children make as grown-ups also affect child investments.

We cast our model in an overlapping generations framework with infinitely-lived dynasties who are altruistic. So in contrast to many models in which parental states are fixed exogenously, and cross-sectional inequality among children is pre-determined once they become adults, current states are decided by all previous members of a dynasty, while current decisions are indirectly affected by all subsequent members of a dynasty (with dynastic discounting). The life-cycle and intergenerational structure, along with multiple stages of human capital investment, complicate notions of equality: parent's own human capital accumulation over the life-cycle happens simultaneously with child investments, but prior to bequest decisions.

We assume complementarity between early investments in the form of parental time,

[7] The other paper focuses more on the quantitative implications of microeconomic foundations for macroeconomic modelling. It combines intergenerational and life-cycle models and shows that having both elements is more consistent with the data, and hence is much more empirical in nature. Here, we make simplifications to bridge quantitative models in general with philosophical concepts.

and later investments in the form of goods. We model the final stage of human capital formation as a college enrolment decision coupled with skill acquisition at college age, as much of inequality can be attributed to differences in educational attainment. The college choice also allows us to separate cross-sectional inequality into differential skill prices and different levels of skill. Post-schooling life-cycle wage growth follows a Ben-Porath human capital accumulation specification. So while educational attainment no doubt has a large impact on human capital, it depends not only on parents' genetics but also parents' income and educational attainment.

The initial distribution over the next generation's human capital and assets are determined by investments in children and bequest decisions, while the terminal distributions of human capital and assets over the parent generation are determined by own human capital investment and life-cycle savings decisions. Since parents make these decisions, we require individual behaviour to be consistent with observing these distributions in a stationary equilibrium, a standard tool in macroeconomics to discipline the distribution of unobserved individual states (human capital). In particular, rationalizing the distribution of human capital jointly with assets is critical, since a framework that can account for earnings persistence but inconsistent with wealth inequality and its transmission would be unconvincing.

We also model conventional forms of government intervention. The data we observe comes from an economy in which the government taxes income progressively to subsidize education, fund social security and assist low income households through welfare payments. For our counterfactual analysis, we focus on shifting transfers between education and welfare.

1.1 Environment

A period is 6 years. There is a unit measure of households whose members live through a life-cycle. where an individual is born at age 0. So each individual goes through 13 periods of life ($j = 0, \ldots, 12$) or 4 stages:

1. Child/College: $j = 0, 1, 2, 3$. Attached to parent, choice to enter the labour market or attain college education (human capital accumulation) at $j = 3$.

2. Young parent: $j = 4, 5, 6, 7$. Independence and bears child at $j = 4$, and continues own human capital accumulation. Simultaneously makes early childhood and subsequent investments in child. Joint college decision for child at $j = 7$.

3. Grandparent: $j = 8, 9, 10$. Continues to accumulate own human capital, and saves for retirement and bequests to child.

4. Retired: $j = 11, 12$. Consumes social security payouts, dies *after* $j = 12$.

Throughout the analysis, primes will denote next generation values.

Fertility is exogenous, and one parent is assumed to give birth to one child at $j = 4$, so $j = j' + 4$ is fixed throughout the lifetimes of a parent-child pair. Until the child's full independence at $j' = 4$, the parent-child pair solve a Pareto problem. These decisions involve consumption, savings, investment in human capital, and college. The family decides whether the child should attend college when he becomes 18 years old ($j' = 3$). The direct cost of college is fixed at κ, but there is also an implicit cost of the child's forgone earnings by delaying entry into the labour market. In retirement, the grandparent consumes his savings and social security benefits, and dies at age 78. He also sets aside a bequest b' for his child at $j = 11$, which is transferred to his child at $j = 12$. The sequence of events is depicted in Figure 1.

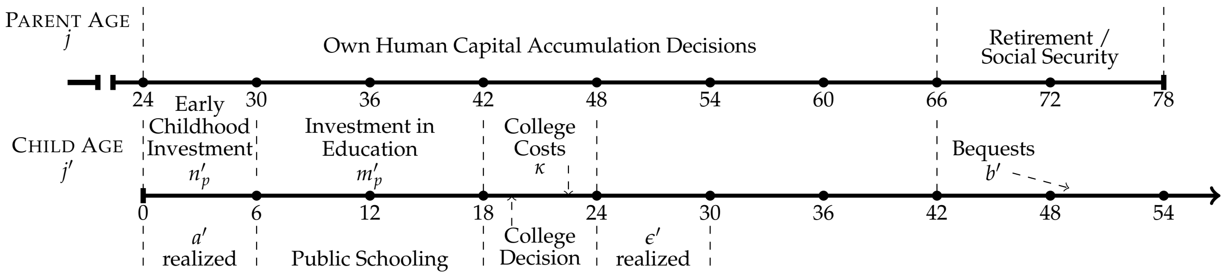


Figure 1: Sequence of Events

1.2 Human Capital Formation

The crux of our model-based approach is the elabourate human capital formation process. Previous studies analyse EOP of the current generation as an endogenous outcome across exogenously defined social background groups, which are proxied by parental circumstances. In contrast, in our model human capital investments occur across infinite generations, so that such circumstances themselves are endogenous outcomes. The only purely exogenous variables are the ability and market shocks, (a, ϵ), respectively. We will later show that this endogeneity of parental types can significantly alter EOP measures.

A child is born with innate ability a', which determines his proficiency at accumulating human capital. The child's ability is stochastic, and depends on the parent's ability. This will capture the "genetic" part of intergenerational transmission. An adult's earnings, or returns to human capital, are subject to a stochastic "market luck shock" ϵ that depends on his own ability, and remains constant throughout his lifetime. This shock captures idiosyncratic risk associated with human capital accumulation, while the dependence on abilities captures both the potential persistence in luck across generations, and a human capital returns premium (or discount) for high ability individuals. Formally, denoting a young parent's ability as a, at $j = 4$ (age 24) he draws (a', ϵ) from a joint probability distribution conditional on a:

$$(a', \epsilon) \sim F(a', \epsilon | a).$$

In addition to exogenous transmissions, we also include human capital accumulation during childhood and adulthood. In particular for the former, we incorporate three important aspects that are well appreciated in the literature: i) parental time and good investments, ii) complementarity between inputs, and iii) children's own investments.

From age 24 until retirement, we assume a Ben-Porath human capital accumulation function,

$$h_{j+1} = a(nh_j)^{\gamma_S} + h_j, \qquad \text{for } j = 4, \ldots, 10, \tag{1}$$

where h_{j+1} is human capital tomorrow and n_j is the time investment in own human capital accumulation. The parameter γ_S depends on whether the adult is college educated $(S = 1)$ or not $(S = 0)$.

During the early childhood of his offspring (ages 0-5, $j' = 0$), a young parent invests n_4 units of time in his own human capital accumulation (on-the-job accumulation of human capital), and spends n_p units of time with his child. When the child goes to secondary school ($j' = 3$), he supports his education with m_p units of consumption goods.[8] The education production function (later childhood human capital formation) is CES in time and goods inputs:

$$h'_3 = \left[\gamma_k^{\frac{1}{\phi}} (m_p + d)^{\frac{\phi-1}{\phi}} + (1 - \gamma_k)^{\frac{1}{\phi}} \left(n_p h_4\right)^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}}$$

where d is a government subsidy, and γ_k captures the expenditure share of education. We have assumed that the time investment in the child, n_p, interacts with the human capital level of his parent, h_4. This is to capture the fact that more educated households (mothers) spend more time with their children (Behrman *et al.* (1999)). The CES parameter ϕ is the

[8]It is not the timing of time and good investments that is germane to our model, but the complementarity between them.

elasticity between parental time and good inputs, which can also be interpreted as complementarity between early and later childhood investments. Cunha *et al.* (2010) estimate this parameter in a model with multiple stages of child investments but only in terms of goods, while Del Boca *et al.* (2014) provide evidence that time is the dominant input when the child is young with its importance declining with age, and that the reverse is true for goods.

From ages 18-23 ($j' = 3$), a child chooses whether or not to attend college, and invests own time n_3' into human capital production according to:

$$h_4' = a' n_3'^{\gamma_{S'}} h_3'^{\gamma_p} = a' n_3'^{\gamma_{S'}} \left[\gamma_k^{\frac{1}{\phi}} (m_p + d)^{\frac{\phi-1}{\phi}} + (1-\gamma_k)^{\frac{1}{\phi}} \left(n_p h_4\right)^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi \gamma_p}{\phi-1}},$$

where $\gamma_{S'}$ depends on whether the child is in college $(S' = 1)$ or not $(S' = 0)$, and γ_p captures the returns to parental inputs. This composite function is intended to capture the fact the child's own time investment becomes more important in later years (Del Boca *et al.* (2012)).

Notice that there are four mechanisms through which human capital is transmitted across generations. First, the ability to *learn* is transmitted across generations, that is, the child's a' is drawn from a distribution that depends on the parent's a. Second, there is partial inheritance of market luck through the correlation of (a, ϵ). This can be viewed as heritable disabilities that affect the ability to *earn*.[9] Third, the resources devoted to child human capital formation, m_p, will be a function of the parent's resources. If capital markets are complete, the investment will be independent of parental resources. However, with capital market imperfections (which we assume), parental resources will matter. Finally, we assume that a higher human capital parent (large h) is better at transmitting human capital beyond his ability to pay for more resources per unit of n_p. While all these channels can be regarded as

[9] In general, $\epsilon' \sim F(\epsilon'|a', \epsilon)$, but we ignore the direct dependence of ϵ' on ϵ because i) it is not clear what a persistence of "luck" means conceptually, and ii) such persistence would not be identifiable without multiple generations of data. The current formulation leaves only one pure channel for genes.

circumstantial and thus not an individual's own responsibility, in the next section we will separate those variables that should be compensated for and those that should not, according to philosophical definitions.

The interaction between child and adult human capital accumulation differentiates us from most previous models, and as we later show, the model can generate empirically observed degrees of intergenerational elasticities between earnings and wealth at different stages of the life-cycle. The assumption that the human capital level of the parent affects both the parent *and* the child is also important. In the data, wages may be thought of as reflecting both innate ability as well as acquired human capital. Viewed this way, it will be affected by unobservable parental (potential) human capital in addition to observables such as parental education or earnings. In particular, the non-linear human capital formation process is pivotal—when complementarity between parental time and goods investments are strong enough, innate abilities may play no role at all.

1.3 Government Policies

We assume there is a government that levies taxes and redistributes subsidies. Our counterfactual experiments will focus on altering the parameters of the latter, which includes a lump-sum transfer g and an education subsidy d too all individuals.[10] As is common in life-cycle earnings models, we also include a social security program in the background.

We first describe the tax system. Denote by e_j the earnings in any working period j. Then, e_j is taxed at a progressive rate $\tau_e(e_j)$, and also subject to a flat rate payroll tax τ_s that is used to fund the social security benefits. The final after tax, after subsidy net earnings of

[10]The rest would capture all other government expenses that do not directly affect the utility of domestic agents; metaphorically, it is "thrown in the ocean."

a working adult is

$$f(e_j) = \left[1 - \tau_s - \tau_e(e_j)\right] e_j. \tag{2}$$

Capital income is taxed at a flat rate τ_k, so we can define $\tilde{r} = [1 + (1 - \tau_k)r]^6 - 1$, the 6 year compounded effective interest rate faced by a household, where r is the pretax annual interest rate.[11]

Social security benefits p are modelled as a function of $\bar{e}$, the average lifetime earnings from ages 24-65 ($j = 5$ to 10).[12] We model it as an affine function:

$$p = p(\bar{e}) = p_0 + p_1\bar{e}, \qquad \bar{e} = \sum_{j=5}^{10} e_j \Big/ 6$$

where (p_0, p_1) are parameters governing the social security regime. Social security benefits are not subject to any tax. The parameters of the program will later be chosen to resemble the actual U.S. social security system.

We assume that the initial distribution F_0 for (a', a, ϵ) is the stationary distribution of $F(a', \epsilon | a)$. The steady state equilibrium and budget balance conditions are explained in detail below.

1.4 Household's Problem

We assume that an adult faces natural borrowing constraints for all savings and education decisions, and a non-negative bequest constraint. In the Appendix A, we present the period-to-period structure of the problem, and in Appendix B explain in detail how, given the tim-

[11]Of course a more realistic system would also capture capital income tax progressivity, but to the extent that capital income is barely accrued by households not in the very top percentiles of the income distribution, and that many loopholes exist for capital income for the very wealthy (Gruber and Saez, 2002), the tractability we gain by this assumption should justify what we miss by this assumption.

[12]Social security benefits in the U.S. are based on the 35 years of an individual's highest earnings.

ing of shocks, we can split the life-cycle into two sub-periods: young and old. This simplifies the structure of the household decision problem and makes it suitable for intergenerational analysis. Here we only present this simplified problem, the state and choices of which will later map into our EOP concepts.

Let $y = 4$ and $o = 8$, the 2 nodes in the life-cycle that an individual needs to make decisions. Since there are no other time costs on and after stage $y + 1$, an adult at this stage would simply maximize the present discounted value (PDV) of lifetime income, which admits deterministic decisions which are unaffected by what happens in his old age.[13] Hence the PDV lifetime income is a function of her own education level, ability, market luck, and human capital:

$$z_{h,y+1}(S, a, \epsilon, h_{y+1}) = \max_{\{n_{y+l}\}_{l=1}^{6}} \left\{ \sum_{l=1}^{6} \frac{1}{(1+\tilde{r})^{l-1}} \cdot \left[f(e_{y+l}) + \frac{(2+\tilde{r})p_1}{6(1+\tilde{r})^{8-l}} \cdot e_{y+l} \right] \right\},$$

where $e_{y+l} = w_S h_{y+l} \epsilon (1 - n_{y+l})$ is her labour market earnings at stage $y + l$, and her human capital production and earnings net of taxes $f(\cdot)$ are specified in (1) and (2), respectively. This problem can be easily solved recursively given knowledge of S: the education of the young parent (1 for college and 0 otherwise), a, his ability, ϵ, his market luck shock, and h_{y+1}, his human capital level at age $y + 1$(=5). Hence, the young parent can take $z_{h,y+1}$ as given.

To solve the rest of the household's problem, there are three value functions we need to keep track of: two value functions at stage y, $W_y(\cdot)$ and $V_y(\cdot)$, before and after the young parent decides whether or not to send her child to college, and $V_o(\cdot)$, the value function at stage o, when her child has his own child. The arguments of these functions are the state variables, which form the circumstances of an individual. The choices made based on the

[13] In laymen terms, it means a grandparent does not change his labour supply decisions depending on whether his child was lucky or not in the labour market, or his grandchild turned to be smart or dumb. This does not seem to be such an unrealistic assumption.

states can be viewed as efforts, which, conditional on the state, should be rewarded based on Roemer's criterion. Of particular interest for us are the states of V_y, which are outcomes of parental investments and intergenerational transmissions, a major part of which is $\mathbf{x} = (S, a, \epsilon, h_y)$, the states that determine a young parent's lifetime earnings. These states are also important when we discuss how to apply EOP concepts to our model in the next section.

We assume that a young parent knows already at childbirth whether or not she will send her child to college, i.e., the value of S' is known.[14] We also assume that she (perfectly) anticipates the present discounted value of bequests, $z_y = b/(1+\tilde{r})^3$, that will be decided by her now old parent (the grandparent). In practice, what this means is that children of rich parents expect to receive a large inheritance later on; vice versa for children of poor parents.

In addition to $(S'; \mathbf{x}; z_y)$, choices are also affected by the child's ability. The young parent makes a joint consumption choice C_y for her and her child, human capital investments (n_p, m_p) in her child and n_y in herself, and carries z_o of wealth into her old age. The child makes his first human capital accumulation decision n_k in the dynamic program:

$$W_y(S', a'; \mathbf{x}; z_y) = \max_{C_y, z, n_y, n_p, m_p, n_k} \left\{ q_y u(C_y) + \beta^4 \int_{a'', \epsilon'} V_o(a''; \mathbf{x}'; z_o) dF(a'', \epsilon' | a') \right\}$$

subject to

$$C_y + \frac{z_o}{(1+\tilde{r})^4} + \frac{m_p}{(1+\tilde{r})^2} = f(e_y) + \frac{z_{h,y+1}(S, a, \epsilon, h_{y+1})}{1+\tilde{r}} + \frac{f(e_k) - S' \cdot \kappa}{(1+\tilde{r})^3} + z_y + G, \quad (3)$$

$$e_y = w_S h_y \epsilon (1 - n_y - n_p) \quad \text{and} \quad n_y, n_p \geq 0, \; n_y + n_p \leq 1$$

$$e_k = w_{S'} h_k (1 - n_k) \quad \text{and} \quad n_k \in \left[2S'/3, 1\right],$$

$$h'_y = a' n_k^{\gamma_{S'}} h_k^{\gamma_p},$$

[14] This means that the young parent *plans ahead of time* to send the child to college, given her own level of human capital, income and wealth, and the revealed ability of her child. This may be a rather strong assumption given that not everything may go as planned, but we need to simplify the life-cycle dimension somewhat to operationalise the intergenerational dimension. Furthermore, while included for completeness, we will show that the college margin does not play a major role.

$$h_k = \left[\gamma_k^{\frac{1}{\phi}} (m_p + d)^{\frac{\phi-1}{\phi}} + (1-\gamma_k)^{\frac{1}{\phi}} \left(n_p h_y\right)^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}},$$

where q_y adjusts utility to account for adult-equivalent family size and subjective discounting, and (e_k, h_k) denote, respectively, the child's earnings and human capital level at college age. The parent's own human capital in the next period is determined by (1), which, in turn, determines her PDV lifetime income $z_{h,y+1}$. The parameter κ is a fixed cost for college, which is in addition to the cost of forgone earnings incurred by the child remaining in school: if $S' = 1$, the child is committed to spend at least two-thirds of a period, or 4 years, out of the labour market and accumulating human capital. The last term in the budget constraint

$$G = g \cdot \left[\sum_{l=y}^{y+2} \frac{q_A}{(1+\tilde{r})^{l-y}} + \frac{2}{(1+\tilde{r})^3} + \sum_{l=o}^{o+4} \frac{1}{(1+\tilde{r})^{l-y}} \right] + \frac{p_0(2+\tilde{r})}{(1+\tilde{r})^8},$$

is the lifetime PDV of the lumpsum portion of subsidies, where q_A is an adult-equivalent scales that adjusts for family size. The value function when old, V_o, is specified below.

We assume that the grandparent makes bequest decisions before the child decides whether or not to attend college. So given z_y, the young parent-child pair simply choose

$$V_y(a'; \mathbf{x}; z_y) = \max_{S' \in \{0,1\}} \left\{ W_y(S', a'; \mathbf{x}; z_y) \right\}.$$

Hence, the grandparent indirectly controls whether or not his grandchild can go to college by leaving more or less bequests (within his budget constraint).[15] This is in fact the only decision he makes, as his lifetime earnings decision is made when he is young. His states are the ability of his now newly-born grandchild, a'', the lifetime earnings of his child (the young parent) which is determined by $(S', a', \epsilon', h'_y)$, and the wealth he carried over from

[15]In other words, a rich grandparent may leave a large bequest not to help his son, but his grandson.

when he was young:

$$V_o(a'';\mathbf{x}';z_o) = \max_{C_o,z'_y} \left\{ q_o u(C_o) + \theta V_y(a'';\mathbf{x}';z'_y) \right\}$$

$$C_o + z'_y = z_o, \qquad z'_y \geq 0,$$

where the last inequality is the intergenerational non-negative bequest constraint. Whatever is not left as bequests is consumed (C_o), which yields utility to the grandparent discounted by q_o.[16] The parameter θ is the altruism factor, which (for positive values) makes the problem dynastic.

1.5 Firm and Stationary Equilibrium

We assume a standard neoclassical firm that takes physical and human capital as inputs to produce the single consumption good. It solves

$$\max_{K,H_0,H_1} F(K,H_0,H_1) - RK - w_0 H_0 - w_1 H_1,$$

where $R = (1+r+\delta)^6 - 1$ is the competitive rental rate and (K, H_0, H_1) are the aggregate quantities of capital and effective units of labour by skill in the economy, respectively. The inclusion of this stand-in firm is what creates general equilibrium effects that may amplify or dampen the effect of policy changes on the resulting long-run equilibrium.

Let X denote the aggregate state spanning all generations, and denote its stationary distribution by $\Phi(X)$. Let $\Gamma(\cdot)$ denote the law of motion for X, which is derived from the agents' policy functions. In a stationary equilibrium, prices (r, w_0, w_1) solve

[16] Bequests are *decided upon* before the grandchild's college decision, but occurs after the realization of the grandchild's market luck shock and great-grandchild's ability shock. Given the time horizon of a dynasty, it does not seem unreasonable to assume that bequest decisions are not affected by these.

1. Market clearing and stationarity:

$$\Phi(X') = \int \Gamma(X, X')\Phi(dX)$$
$$K = \int_X \left(s_j^* + b^*\right) \Phi(dX) \tag{4}$$
$$w_S H_S = \int_X e_j^* \Phi(dX, S, 3 \leq j \leq 10).$$

   where s_j^* are the optimal savings decisions of an individual at stage j, b^* the bequest decisions of individuals at stage 11, and e_j^* the earnings resulting from the optimal human capital and time investment decisions of adults in stage j.

2. Government budget balance:

$$g = \pi_g \bar{\mathbf{e}}^*, \qquad d = \pi_d \bar{\mathbf{e}}^*,$$

   i.e. the subsidies (g, d) are fixed fractions (π_g, π_d) of average earnings $\bar{\mathbf{e}}^*$:

$$\bar{\mathbf{e}}^* = \int_X e_j^* \Phi(dX, 3 \leq j \leq 10). \tag{5}$$

   The social security regime is also balanced:

$$\tau_s \bar{\mathbf{e}}^* = 2\left[p_0 + p_1 \int_X \bar{e}^* \Phi(dX)\right],$$

   where $\bar{e}^*$ is the past 6-stage average earnings of individuals in stages 11 and 12.

Note that government budget balance is imposed only to discipline the size of the subsidies and social security payments. In our policy experiments, we ignore this condition and keep the government parameters fixed at their benchmark values.[17]

[17] For a fixed set of parameters, the economy converges to a unique stationary distribution regardless of the

2. Calibration

The model is calibrated to 1990 United States. We choose 1990 mainly due to data availability issues, and also to avoid extreme temporal events (such as the oil crisis of the 70s or Great Recession of recent).

2.1 Parametrization

Exogenous processes We assume an AR(1) process for ability shocks:

$$\log a' = -\left[(1-\rho_a)\sigma_a^2\right] \Big/ 2 + \rho_a \log a + \eta, \quad \eta \sim \mathcal{N}(0, (1-\rho_a^2)\sigma_a^2) \tag{6}$$

where σ_a^2 is the unconditional variance of abilities. For luck shocks assume:

$$\log \epsilon' = -\sigma_\nu^2/2 + \rho_\epsilon \left(\log a' + \sigma_a^2/2\right) + \nu, \quad \nu \sim \mathcal{N}(0, \sigma_\nu^2),$$

so both abilities and luck are assumed to have a mean of 1. Abilities and luck are discretized into 4 and 3 grid points, respectively, using the Rouwenhorst method (Kopecky and Suen, 2010).

Preferences and Technology First, the parameters (β, v) are found in equilibrium so that we hit exactly an annual interest of $r = 4\%$ and a college earnings premium of 46.8%, which is computed from the 1990 IPUMS CPS. In experiments, we fix (β, v) to their calibrated values and find the interest rate r that clears the asset market and wage ratio w that clears

initial distribution if Γ is ergodic. As is the case with most similar quantitative models that extend Aiyagari (1994), we cannot prove uniqueness analytically, but have checked numerically that the distribution monotonically converges to a unique point.

the labour market by skill. Period utility is modelled as a standard CRRA utility function,

$$u(c) = (1 - \theta\beta^4) \cdot c^{1-\chi} / (1 - \chi),$$

for all individuals, including children and retirees. The normalization by $(1 - \theta\beta^4)$ facilitates interpreting the value function in terms of consumption equivalent values. The aggregate production function and capital stock evolution are parametrised as:

$$F(K, H_0, H_1) = K^\alpha H^{1-\alpha}, \qquad H = \left[v^{\frac{1}{\sigma}} H_1^{\frac{\sigma-1}{\sigma}} + (1-v)^{\frac{1}{\sigma}} H_0^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}},$$
$$K' = (1 - \delta)K + I.$$

This parametrisation is consistent with Heckman *et al.* (1998), and we use their point estimate for the elasticity between skilled and unskilled labour, $\sigma = 1.441$.

The parameters for human capital production are calibrated within the model, except for ϕ. We set the elasticity of substitution between age 6 and 18 human capital investments $\phi = 2.569$ as estimated in Cunha *et al.* (2010).

Policy Parameters Tax rates should capture marginal tax rates, not average tax rates. Hence, τ_k is set as 0.31, the 1990 value from Gravelle (2007)'s study on effective marginal tax rates on capital income. For labour income taxes, we follow the log specification of average taxes in Guner *et al.* (2014), which they show retains attractive properties for the marginal tax rate:

$$\tau_e(e) = \tau_0 + \tau_1 \log (e / \bar{\mathbf{e}}),$$

where again, $\bar{\mathbf{e}}$ is the average earnings in the economy. We set (τ_0, τ_1) to the values estimated in their study, and calibrate $\bar{\mathbf{e}}$ to the average earnings in our model economy.[18]

We then use $\bar{\mathbf{e}}$ to compute transfers and college costs. Parameters for the social security system, (p_0, p_1, τ_s), are set as follows. First, we fix $p_1 = 0.32$, the median replacement rate for social security payments. Given p_1, we set (p_0, τ_s) to balance the social security budget and match an average replacement rate of 40%, as reported in Diamond and Gruber (1999). To simplify the numerical procedure, we then assume that earnings in model periods $3-4$ (ages 18-29) are negligible in the aggregate so that $\bar{\mathbf{e}} \approx \int_X \bar{e}\,\Phi(dX)$, where $\bar{e}$ was individual average earnings from ages 30-65. Given this assumption, $\tau_s = 0.1$ and $p = \pi_p \bar{\mathbf{e}}$ with $\pi_p = 0.133$.

Transfers as a fraction of average earnings, (π_g, π_d), are set to $(2\%, 5/(1-\alpha)\%)$, respectively. We view lump-sum transfers mainly as welfare for the poor, and the size of welfare transfers in the U.S. was approximately 1-2% of average earnings throughout the late 1980s to mid 1990s. Education transfers in the model are assumed to be public spending on secondary education and below in the data, which is obtained from the 1990 Digest of Education Statistics. As a fraction of GDP, this value is 5%, and the division follows from the labour income share of total output, since π_d is a fraction of average earnings.

Annual college costs are set to equal $\pi_\kappa = 30\%$ of average earnings, which is roughly equal to the ratio of college costs from the National Center of Education Statistics and average earnings from the CPS throughout the 1980s-1990s, so that

$$\kappa = \pi_\kappa \bar{\mathbf{e}} \cdot \left(1-\beta^4\right) \Big/ \left(1-\beta^6\right).$$

Since all policy parameters are expressed as a fraction of $\bar{\mathbf{e}}$, we solve the fixed point problem

[18] Although their estimates are based on 2000 IRS tax returns, the 1990s displayed significantly modest changes to tax policies compared to other decades.

Parameter	Value	Description
χ	2	CRRA coefficient, $u(c) = \frac{c^{1-\chi}}{1-\chi}$
ϕ	2.569	CES age 6 vs 18 human capital investments, Cunha *et al.* (2010)
σ	1.441	CES high vs low skill labour, Heckman *et al.* (1998)
(α, δ)	(0.322, 0.067)	capital income share / depreciation rate, Huggett *et al.* (2011)
(τ_0, τ_1)	(0.099, 0.035)	earnings tax constants $\tau(e) = \tau_0 + \tau_1 \log \frac{e}{\bar{\mathbf{e}}}$, Guner *et al.* (2014)
(q_A, π_g)	(1.7,0.02)	adult equivalence scale, lumpsum subsidies as fraction of average earnings $\bar{\mathbf{e}}$
(π_d, π_κ)	(0.05, 0.3)	education subsidies and cost of college, as fraction of average earnings $\bar{\mathbf{e}}$
(p_1, π_p, τ_s)	(0.32,0.08,0.12)	social security parameters implied by balanced social security budget and median replacement rate of 40% (Diamond and Gruber (1999)), refer to text.
$(\bar{r}, \overline{EP})$	(4%,46.8%)	rate of return on capital / earnings premium in 1990 IPUMS CPS

Table 1: Fixed Parameters

(5) such that in equilibrium, individual choices given the implied values of tax, subsidies and college costs leads to the average earnings used to compute those values.

Lastly, q_A, the adult equivalence scale for households with children, is set to 1.7, which is the adjustment factor for two-adult households with two children versus those without children, used by the OECD.

Setting Prices The parameters (β, v) are found in equilibrium as follows. The firm's profit maximization implies that

$$RK = \alpha F, \qquad WH = (1-\alpha)F, \qquad W = (1-\alpha)\left(\alpha/R\right)^{\frac{\alpha}{1-\alpha}}$$

where

$$W = \left[v w_1^{1-\sigma} + (1-v) w_0^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \tag{7}$$

In the benchmark calibration, we choose the discount factor β to imply an annual equilibrium interest rate of $\bar{r} = 4\%$, while in experiments we fix β to its calibrated value and choose r to clear the asset market. Since we do not directly observe skill prices in the data, we set (w_0, w_1) to match the average share of college-educated individuals. The firm's cost minimization implies the equilibrium wage ratio

$$w \equiv w_0/w_1 = \{[(1-v)H_1]/(vH_0)\}^{\frac{1}{\sigma}},$$

and since we observe EP, the college earnings premium in the data, and also L_1, the population share of individuals with college education, wages must satisfy

$$\frac{H_1/L_1}{wH_0/(1-L_1)} = \left(\frac{v}{1-v}\right)^{\frac{1}{\sigma}} \left(\frac{H_1}{H_0}\right)^{\frac{\sigma-1}{\sigma}} \cdot \frac{1-L_1}{L_1} = EP$$

$$\Rightarrow \quad \frac{H_1}{H_0} = \left[\frac{1-v}{v} \cdot \left(\frac{L_1 EP}{1-L_1}\right)^{\sigma}\right]^{\frac{1}{\sigma-1}}, \tag{8}$$

$$\Rightarrow \quad w = \left(\frac{1-v}{v} \cdot \frac{L_1 EP}{1-L_1}\right)^{\frac{1}{\sigma-1}}. \tag{9}$$

Using (w, W) from equations (7) and (9), we can set (w_0, w_1) for any given (r, v, EP, L_1). In the calibration, we choose v to match an earnings premium of $\overline{EP} = 46.8\%$ and college enrolment rate of $L_1 = 41.3\%$, which we compute from the 1990 IPUMS CPS. In experiments, we fix v to its calibrated value and find the wage ratio w that clears the labour market by skill.

Simulated Method of Moments In sum, 16 parameters are taken from the literature or exogenously fixed by the data, see Table 1. As explained above, three parameters, $(\beta, v, \bar{\mathbf{e}})$, are found in equilibrium. The remaining 9 parameters are chosen by using the model to simulate 9 equilibrium moments to match an exactly identified number of empirical moments. Specifically, the parameter vector is

$$\Theta = [\gamma_0 \; \gamma_1 \; \gamma_p \; \gamma_k \; \rho_a \; \sigma_a^2 \; \rho_\epsilon \; \sigma_v^2 \; \theta]'$$

and the vector of empirical moments, M_s, is summarized in Table 2. Moments 1-3 and 5 are

$M_{s,i}$	Description	value
i =1.	Average wage ratio at ages 48-53 vs. 30-35, $S = 0$	141%
2.	Average wage ratio at ages 48-53 vs. 30-35, $S = 1$	154%
3.	College enrolment rate	41%
4.	(Secondary and below) education expenses-GDP ratio	7%
5.	Education persistence $P(S' = 1 \mid S = 1)$ (refer to text)	63%
6.	IGE of lifetime earnings	0.60
7.	Gini coefficient of lifetime earnings	0.49
8.	Gini coefficient of retirement wealth	0.62
9.	Fraction of population receiving zero bequests	45%

Parameter	γ_0	γ_1	γ_p	γ_k	ρ_a	ρ_ϵ	σ_a	σ_ν	θ
Value	0.63	0.74	0.97	0.64	0.35	-0.55	0.17	0.52	0.57
Chosen in	β	v							
Equilibrium	0.94	0.51							

Table 2: SMM Moments and Calibrated Parameters

The probability measure $P(\cdot)$ is taken over the equilibrium stationary distribution.

computed from the NLSY79. College, $S = 1$, is defined as at least one year of education beyond high school graduation, and high school graduates or below are defined as $S = 0$.[19] To obtain the persistence of education we refer to the Children of NLSY79, from which we can compute $\mathbb{P}(S' = 1 \mid S = 1)$ in the sample.

The education expense ratio is from the 1990 Digest of Education Statistics. Note that this is the aggregate of both public and private expenses. The intergenerational elasticity (IGE) of lifetime earnings is from Mazumder (2005) using data from the SER. The number 0.6 is the coefficient obtained by running a regression on the logarithm of fathers' earnings averaged over the years 1970-1985, where the dependent variable is the logarithm of children's earnings (both sons and daughters) averaged over the years 1995-1998. The Gini coefficient of lifetime earnings is from Leonesio and Bene (2011). Their data set is based on Social Security Administration data from 1980-2004, for more than 3 million people. The number we take is the Gini coefficient of a 12-year present discounted value of earnings using data on men aged 31-50 in 1993. The retirement wealth Gini is computed in Hendricks (2007) using

[19]Hence, we make no distinction between high school dropouts and high school graduates, nor college dropouts, some college, college graduates and beyond. Increasing the education categories would perhaps make the study more interesting, however the numerical analysis becomes exponentially costly.

	IGE of $x(j_p, j_k)$:		$WPP = 1$
	$e(6,5)$	$s(9,6)$	
Data	0.528	0.365	0.293
Model	0.513	0.341	0.326

Table 3: Non-targeted Moments

data from the PSID. McGarry (1999) documents detailed information on intergenerational transfers using data from the HRS and AHEAD. The AHEAD survey elicits the subjective probability of leaving a bequest, of which the sample average is 0.55.[20]

We find the point estimate $\hat{\Theta}$ by solving

$$\hat{\Theta} = \arg\min_{\Theta} \left[M(\Theta) - M_s\right]' \left[M(\Theta) - M_s\right], \tag{10}$$

where $M(\Theta)$ are the simulated model moments. The resulting benchmark parameters are summarized in the top panel of Table 2. We match all moments almost exactly. Details of how the model is solved for numerically are outlined in Appendices C-D.

2.2 Properties of the Benchmark Model

While our main goal is to operationalise EOP concepts, we briefly discuss properties of the benchmark model and its performance.

Non-targeted moments In Table 3, we compare non-targeted moments with moments that are available from other datasets. In the first two columns, to compare with previous empirical studies that use data from parent and children at different ages, we show IGE's of earnings and wealth when using numbers from different periods of the parent and child's life-cycle. The first column shows the IGE of earnings using parental earnings in period 6 and child earnings in period 5. This is notably lower than our 0.6 in the benchmark us-

[20] In comparison, 43% of respondents in the HRS respond that affirmatively to the question on whether they "expect to leave a sizeable inheritance."

ing lifetime earnings, and remarkably similar to the magnitude found in Mazumder (2005) when using 6-year averages of earnings.[21] Since our model has no idiosyncratic earnings shocks following the once-and-for-all luck shock in period 4, the different number comes purely from life-cycle effects. Next, we compare wealth holdings at parent period 9 and child period 6. Again, the magnitude is very close to what is found in Charles and Hurst (2003) who compute the IGE of wealth for parents around the age of 50 and children around the age of 37 years old. While not shown in the text, we have verified that for both earnings and wealth, the IGE's become larger when the age of the parent and child are closer to each other.

In the last column, we compare the fraction of households who participate in welfare programs ($WPP = 1$) in the model to the fraction of families who receive AFDC in the NLSY79, as tabulated by Gottschalk (1990)). In the Gottschalk sample, AFDC receipts comprise approximately 10% of the participating family's total income. Our model does not have any welfare transfers per se, so instead we define $WPP = 1$ households as those for whom the lump-sum transfer, g, exceeds 10% of the parent's earnings in period 5, when the child is 6-11 years old. The fraction of WPP households thus defined is very much in line with Gottschalk's numbers.

In summary, we believe that the model not only matches the moments that we are explicitly targeting, but also moments of interest that are not directly targeted by the calibration procedure.

Childhood human capital production The parameters (γ_p, ϕ) play an important role, as they capture how much parental investments matter for education. If γ_p is high, the parent may need to sacrifice own human capital accumulation to educate the child, and even more

[21] Most children in his study were in their early 30s, so period 5 is a reasonable proxy. For parents, he looks back 10-15 years, so that the children are of high school or college entry age. Although period 6 is slightly lower than the average age in his sample, this is when the child is of high school/college age.

	Benchmark	i. $\phi/2$	ii. $\gamma_p = 1$	iii. $\rho_a = 0$	iv. $\rho_\epsilon = 0$
Enrollment rate (%)	41	-6.3	+15.2	-1.8	-2.8
$P(S' = 1 \vert S = 1)$ (%)	63.4	+8.7	-13.3	-9.7	-1.2
IGE lifetime earnings ($\times$100)	60.0	-1.2	-41.9	-1.1	+1.7
Lifetime earnings Gini ($\times$100)	48.8	+2.5	-11.3	-0.6	+3.1
Retirement wealth Gini ($\times$100)	62.0	+3.9	-9.8	-0.8	+2.2

Table 4: Effect of Childhood Human Capital Parameters
Columns i-iii are the change in moments from their benchmark values.

so when time and goods are less substitutable ($\phi \to 0$). The importance of early human capital is what will drive our results later. Here, we stipulate the long-run consequences of key parameters.

When time and goods investment become less substitutable (row i in Table 4), fewer children make it to college and educational attainment becomes more persistent. This is because only richer parents are able to send their children to college: while all parents are endowed with time, poor parents are not able to match their time investments with goods. However, this has a small impact on lifetime earnings persistence because the larger goods investments made by parents are offset by the decline in time investments, which plays a more direct role in children's human capital formation (since the returns to parental time investment is increasing in the parent's human capital, which determines earnings).

The case when $\gamma_p = 1$ is when parents have the largest impact on children's lifetime earnings (row ii). All children reach higher levels of human capital and more make it to college. More individuals start their working lives with high levels of human capital, and many parents achieve the optimal level of investment in children, reducing the IGE. This is the ideal world in which persistence and cross-sectional inequality both decrease.

Exogenous transmissions In contrast, as long as human capital investments are in place, the persistence of learning abilities, ρ_a, plays a small role (row iii). There is some shuffling in terms of college enrolment, as the drop in its persistence is much larger than the drop in overall enrolment. This means that more poor parents, and less rich parents, send their children to college. Nonetheless, even with no persistence in abilities, the long-run drop in the IGE is negligible; the same is true to both the earnings and wealth Gini indices.

Put together, this implies that whether or not children make it to college plays a small role at most in terms of intergenerational persistence. When early human capital investments become more difficult (row i), the IGE does not change despite more persistence in college enrolment; the opposite is true when abilities are uncorrelated. That is, college is more an outcome of selection than a driver of inequality. It also implies that the early investment channel plays a larger role than exogenous transmission of abilities. This will important for our results when discussing EOP concepts.

It is also of interest to investigate the role of ρ_ϵ, the persistence of learning abilities across generations and the correlation between ability and luck shocks. Below, we will consider the learning abilities of one's parent as circumstantial. Since the correlation between one's own abilities and luck is calibrated to be negative, then, there is less inequality among children conditional on the parents' abilities than when there is no correlation.[22]

When we set $\rho_\epsilon = 0$, there is a slight increase in the IGE of lifetime earnings (row iv). Remember that the benchmark value for ρ_ϵ is negative, pushing lifetime earnings toward the mean. Educational attainment drops and also its persistence. This means that there are less parents who send their children to college in anticipation of a high luck shock, most of whom were from poorer families. Without the mean reverting luck shock conditional on ability a that happens *after* labour market entry, both earnings persistence and cross-sectional inequality increase. Conversely, this means that this correlation acts as a built-in equalizer that partially compensates for one's circumstances.

[22]We discuss this more in Appendix F.

3. Operationalizing EOP

Previous EOP concepts focus on the distribution of an outcome variable Y. This is viewed as the outcome of (relative) efforts E that an individual is solely responsible for, and therefore be rewarded, and heritage or background types B that an individual played no role in shaping, i.e. circumstances that an individual cannot be held responsible for (Roemer, 1993). More recently Vallentyne (1997, 2002) theorized how luck should be rewarded, arguing that some types of luck should be (relatively) rewarded (L_e) while some should not (L_b). Formally, outcome Y is then defined as a function:

$$g : (E, B; L; \Omega) \mapsto Y, \qquad L = [L_e \; L_b] \tag{11}$$

where Ω is the environment which can also be shaped by a planner. It is implicitly assumed that this function is known, in particular how it may respond to the environment Ω: this is also true in our model. Since (E, L) are difficult to observe in the data, EOP has been discussed via means of the conditional distribution $F(Y|B)$. In a model without luck, Roemer (1993) argues that although justice requires that these conditional distributions should be equal, it is infeasible to achieve this so that some measure (median, mean, or quantile) be equalized. Lefranc *et al.* (2009) presents how to operationalize such concepts empirically, cautioning that luck cannot be separated from effort in the data.

Our goal is not to define a new concept but to operationalise these concepts in our model. For this, we need to decide on what corresponds to (Y, E, B, L_e, L_b). For what follows, a variable with the subscript -1 denotes the variable of the previous generation.

3.1 Luck

One advantage of having a model is that we know exactly what L_b is as opposed to L_e. Our model has two sources of luck, genetic ability a and market luck ϵ, which are exogenous to any individual choices by construction. Most authors agree that own ability should be rewarded (Vallentyne, 1997; Arneson, 2010). Otherwise, it could violate other ethical values, such as the libertarian value of self-ownership of natural endowments. However, having a high ability *parent* does not constitute self-ownership. In other words, only the part of ability that is uncorrelated with the parent should be rewarded.

The market luck shock can be interpreted as Dworkin (1981)'s notion of "brute luck", in particular "later brute luck" (Vallentyne, 2002) since it occurs in adulthood. Notice that conditional on parental ability, all individuals face the same amount of risk: parental ability is the sole source of *ex ante* inequality in terms of market luck. So according to (Vallentyne, 2002; Arneson, 2012), what should be compensated for is *not* any of these shocks per se, but the ability level of the parent.

This is not the end of the story. In our intergenerational setup, we need to consider not only the luck of the previous and current generations but also the next. Should an individual be rewarded or compensated for having a high ability child, or child with great luck in the market? If we are consistent in how we separated the luck between generations $t-1$ and t, it becomes clear that there is no need to compensate generation t for whatever happens in generation $t+1$: the correlation between generations t and $t+1$ come solely through a, which is a natural endowment, and any shocks to generation $t+1$ are then "later brute luck."

In summary,

1. $F(a, \epsilon | a_{-1}) \sim L_e$: conditional on parental ability, all luck should be rewarded relatively;

2. $(a_{-1}, \epsilon_{-1}) \sim L_b$: parental luck should be compensated for;

and note that 1. includes luck occuring to one's offspring as well.

3.2 Outcomes and Efforts

While it was rather straightforward to separate luck by construction, mapping (Y, E, B) into our model is less obvious because these variables are endogenous in our model. This also raises some insight into intergenerational concerns that have not received much attention in the applied literature.

In philosophical applications investigating what constitutes EOP and economic models that focus on childhood,[23] the wage processes or outcomes of the parent generation are taken as exogenous and children's outcomes considered final. What gives us a novel perspective on EOP is that these processes themselves are the outcomes of efforts of previous generations, and that the choices of the child generation forms the bases for the grandchild generation. Formally, in (11), (E, B) themselves are in fact (assuming that Ω remains constant)

$$B \equiv B(E_{-1}, B_{-1}, L_{-1})$$

while although E is in principle also functions of previous generation variables, since it is relative given B we can assume this has been already accounted for. However, even then, such efforts would not only depend on one's self but also their offspring, so

$$E \equiv E(E', L').$$

Under a dynastic view, this raises the concern that if B is compensated away, previous generations' efforts are not being rewarded (Swift, 2004). Conversely, if we take an individualis-

[23] E.g., Caucutt and Lochner (2012); Del Boca *et al.* (2014).

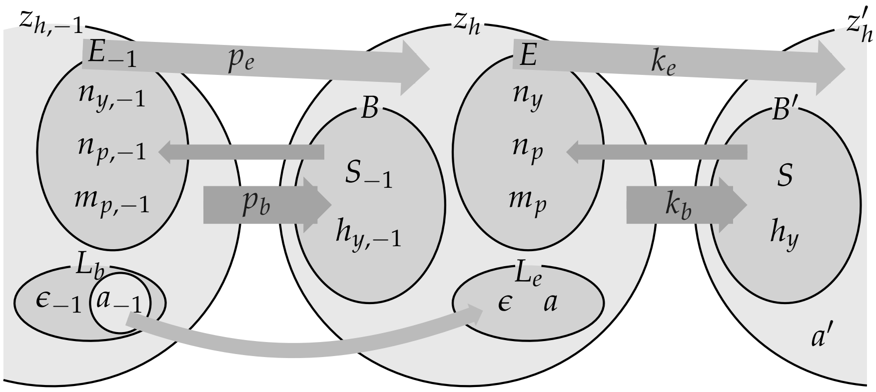


Figure 2: Overlapping generations of luck, background and effort.

Part of outcome Y is formed by parental efforts E_{-1} through p, which itself depends on B. Furthermore, B depends on Y_{-1}, which includes previous generation's efforts. HenceIf p is not rewarded then, k should be compensated for. The only luck that should be compensated is parental ability, a_{-1}. All other outcomes are either "later brute luck" in the sense of Vallentyne (2002), or natural endowments that constitute the individual.

tic view and ignore the fact that B at least partially reflects previous generations' efforts, the part of $g(E, \cdot)$ that is affected by future generations should not be considered. In other words, previous generations should be compensated for what current generations are compensated for, or if not, current generations should not be punished for what previous generations are not rewarded for. This is best depicted graphically in Figure 2: if the p-links are ignored, so should the k-links, and vice versa. Note that we are not advocating whether or not such links should be ignored, we are arguing for logical consistency.[24] In our quantitative experiments, we consider both stances and show that they can have vastly different implications.

Let us formalize the graphical depiction in Figure 2. An advantage of our model-based approach is that we can easily compare the states and choices of the young and old age adults: all variables have clear interpretations, from which the authority making ethical judgments can choose. Choices that occur from age 24 onward constitute effort, which, conditional on B, form the effort variables E. For such choices, then, the individual should be rewarded according to Roemer's EOP definition. It remains to determine B.

Since abstract discussions as well as previous empirical studies have focused on income

[24] In reality, the interdependency of states and choices are much more complex than the few linkages we depict in Figure 2; here we only depict some of the representative ones.

or earnings, let us first inspect the lifetime earnings of a young adult in (3) as a potential candidate for Y. This is possible since earnings are observable in the data. Ignoring $f(e_k)$, which is earned by the child and not the parent, and also time allocation choices which is part of E, the primary determinants of earnings are $\mathbf{x} = (S, a, \epsilon, h_y)$: one's education level, ability, market luck, and human capital level upon independence. We already argued above that a_{-1} and not (a, ϵ) should be compensated for. Own (S, h_y), in turn, depend on both parental investments, which are part of E_{-1}, and the parent's $(S_{-1}, h_{y,-1})$. This is represented by p_b and B in Figure 2, respectively.

At first glance, it seems that B should be equalized away according to Roemer's notion of EOP. However, E_{-1} is a function of B, so then the parent would not be rewarded for efforts that he made, not for himself, but for his child. This is the direct linkage, p_e. Furthermore, B is also partially a function of the previous generation's efforts through Y_{-1} and p_b. Then, full compensation would violate the principle that efforts—not of the current generation, but of the previous generation, and the one before that, and *ad infinitum*— should be rewarded. This is the indirect linkage.

Under a dynastic view, all generations' efforts should be rewarded. Then there is no role for B: it can be expressed as an infinite regress of genetic and brute luck. The only element of Y that should be compensated for is whatever inequality that stems from $L_b \sim (\epsilon_{-1}, a_{-1})$ alone. The problem in practice is that such infinite histories are not observed, but our model offers a solution. Since it is solved so that intergenerational investments are consistent with a long-run equilibrium, we can consider whether a far away ancestor is rewarded for his dynastic efforts and examine EOP accordingly.

Besides practical limitations, one could take an individualistic view and argue that far-away generational efforts should not be rewarded, in which case B should still remain a primary concern (Arneson, 1998). That is, dynastic concerns should not be included when

Parent Type	Child's Outcome Distribution		
(quartiles)	z_h	z_w	c_v
$z_{h,-1}$	Case 1	Case 3	Case 5
a_{-1}	Case 2	-	Case 4

Table 5: Types and outcome distributions considered.

considering justice for individuals. But if a planner decides to ignore the p-linkages, the outcome variable, Y, should also be purged of k-linkages to maintain a consistent application of redistributive justice across generations.

In laymen terms, if at least part of the motivation of an individual to achieve a higher social status was to better the prospects of his offspring, this should be rewarded. Otherwise, when comparing incomes of the child generation, one should first deduct the education expenses the child incurs for the *grandchild* generation. To take all this into consideration, all EOP measures are analysed by examining the conditional distributions of three objects. The first two are

1. z_h: children's lifetime earnings, which is observable in the data and has been the conventional object of analysis in previous studies.

2. $z_w \equiv z_o - z'_y$, i.e., implied wealth at age 42 less bequests to be left for the grandchild. This is free of investments the child has made in the grandchild, both in terms of human capital and assets.

Our structural model allows us to do more. Since individuals in our model are rational, the reason they invest more or less in their children is solely because they derive utility from doing so. The continuation utility of the child factors in all costs and benefits that come from previous and subsequent generations. So by comparing the utilities to the financial measures in items 1 and 2 conditional on $z_{h,-1}$, which subsumes efforts from previous generations, to when we instead condition on a_{-1}, we can analyse how such efforts are being rewarded *dynastically*. Indeed, it turns out that in our model, most of such efforts manifest themselves as utility gained from future generations (but at a decreasing rate), not one's own consumption. So specifically, our third object is

3. $c_v \equiv \left[(1-\chi)V_y\right]^{\frac{1}{1-\chi}}$: to make the units comparable with z_h and z_w, we transform the variable into consumption equivalent units.

The types that are chosen for conditioning are based on either: i) parental $z_{h,-1}$ (lifetime earnings) quartiles, which is a one-dimensional stratification of $\mathbf{x}_{-1}$ and also applicable to real data, or ii) the four discrete points of a_{-1}, parental ability, which is not observed in the data.[25] These cases are labelled 1-5 as shown in Table 5. If cases 2. and 3. closely mirror cases 4. and 5., respectively, we can conclude that factoring in the intergenerational investments brings us closer to utility measures of outcomes, rather than dollar outcomes.

3.3 Ordinal Measures of EOP

Given the numerically simulated stationary distribution Φ, we compute both ordinal and cardinal measures of EOP. The ordinal measures are developed in Lefranc *et al.* (2008, 2009) and applied to the variables discussed above. Their goal was not to propose a new concept for EOP, but to operationalise how concepts such as Roemer's would be applied to real data. We simply apply the same methods to our simulated data, but to a wider range of variables as discussed above. For each of the 5 cases, we examine the following for each pair of conditional distributions:

1. conditional C.D.F.'s and Lorenz curves (for visual inpsection)

2. Equality and stochastic dominance (FSD,SSD) tests

[25]Of course in theory we can increase the number of types to as many as we wish, but we keep the number small so that the comparisons are easily comprehensible. In particular, note that we are ignoring conditioning on ϵ_{-1}.

3. Lorenz dominance tests

Here we give a brief explanation of items 2.-3.; we refer the interested reader to Lefranc *et al.* (2008) for details. Item 2 is conducted as follows:

1. Test whether we can statistically reject that the distributions are equal using a two-sample Kolmogorov-Smirnov test. If we cannot, EOP is supported in the strong sense.

2. If not, test SSD for the two distributions. Check, statistically, whether neither dominates the other or both dominate each other. If they do, EOP is supported in the weak sense. For checking stochastic dominance, we simulate p- values via recentered bootstrap according to Barrett and Donald (2003).

Note the hierarchy of these tests. The first test amounts to saying that not only are different types compensated for, but also relative efforts and luck are equally rewarded within each type. This corresponds to an ideal notion of EOP that even in a simple stylized setup as in Roemer (1993) cannot be expected to be feasible. For the second test, there is a sense that the case when both dominate each other is "more equal" than when neither dominate each other. However, given that the first test rejects equality, there is no formal method to rank the two outcomes. The FSD test only applies when EOP is violated, meaning one distribution dominates the other at the second-order; if in addition it dominates at the first-order it implies a strong violation of EOP since one type would be preferred to the other type for *any* unobserved effort-luck combination. The Lorenz dominance test also only applies when EOP is violated. A Lorenz curve normalizes all outcome variables so that a mean effect is removed, revealing whether comparable effort-luck combinations in the sense of Roemer (1993) are being equally rewarded, conditional on circumstances.

3.4 Cardinal Measures of EOP

Since the above measures are all ordinal (qualitative) and binary (equal or unequal), we also compute cardinal measures in order to quantify IOP. Since policy changes shift the entire stationary equilibrium, this is especially conducive for quantifying the effects of counterfactual policies. We somewhat deviate from previous studies and use a normalized Theil index for a cardinal comparison. While a Gini coefficient or similar variants may be conceptually closer to stochastic dominance tests, we opt for the Theil index because it is readily decomposable into a Theil index across the mean of each type, plus the weighted average of Theil indices within types. Hence it is ideal for giving a sense of whether inequality stems from inequality across types or unequal compensations across effort-luck combinations within types. For comparison, we also compute mean logarithm deviations (MLD) in Appendix Table E9, which has been used more commonly in the literature (Roemer and Trannoy, 2015).[26]

Besides this, the Theil index, which is defined as maximum minus observed entropy, is ideal for positive economic analysis. Grünwald and Dawid (2004) have shown (subject to some regularity conditions) that the problem of maximizing entropy is equivalent to minimizing worst-case expected loss. Hence, an unconstrained Rawlsian planner would be equivalent to an entropy maximizer. Then, similarly to Sen (1974, 1976)'s social welfare function that considers both average incomes and the Gini coefficient, one could consider a mixed utilitarian-Rawlsian planner that maxi-mins a utilitarian objective function subject to an entropy constraint. The attractiveness of such an approach is that the planner's concern for equality can be represented by a constant parameter, which the economist can take as given, and that a dynamic decision problem can be represented in a simple recursive form

[26] Much of the previous literature uses MLD, especially when the outcome variable is income. Both the Theil index and MLD are special cases of the Generalized Entropy Index, of which desirable properties are relativeness and decomposability. We choose the Theil instead of the MLD since one desirable property of MLD is that under certain conditions, it is equivalent to the Theil index (Ferreira and Gignoux, 2011).

(Hansen and Sargent, 2007).

Suppose there are J types indexed by $j = 1, \ldots, J$, each of mass μ_j, so total population is $\mu = \sum_{j=1}^{J} \mu_j$—$\mu$ does not have to equal 1. Denote the conditional distribution of outcomes, y, within each type by the c.d.f. F_j. The Theil index in this setup is

$$T = \sum_{j=1}^{J} \frac{\mu_j}{\mu} \cdot \int \frac{y}{\bar{y}} \cdot \left(\log \frac{y}{\bar{y}}\right) dF_j(y) = \sum_{j=1}^{J} s_j \left[\frac{\bar{y}_j}{\bar{y}} \cdot \left(\log \frac{\bar{y}_j}{\bar{y}}\right) + \int \frac{y}{\bar{y}_j} \cdot \left(\log \frac{y}{\bar{y}_j}\right) dF_j(y)\right]$$
$$= T_b + \sum_{j=1}^{4} s_j T_j$$

where

$s_j =$ type j's share of of total y in the population,

$\bar{y} =$ mean of y in the population,

$\bar{y}_j =$ conditional mean of y in type j,

$T_b =$ Theil index across the means of each type, and

$T_j =$ Theil index within each type.

Then the problem of maxi-mining the expected outcomes across groups as in Roemer (1993) is equivalent to minimizing T_b. More objectively, given a total level of inequality T, EOP can be quantified by the contribution of between-type inequality T_b/T. Furthermore, by cross-examining the T_j's of each type j, we can also compare the extent to which similar relative efforts are being rewarded similarly.

We also compute the change in a few simple aggregate statistics as is typically done in the economics literature: i) the IGE, or intergenerational elasticity of lifetime earnings, and ii) $\Delta \log \bar{y} = \log \bar{y} - \log \bar{y}_B$ where $\bar{y}$ is the mean of $y \in \{\bar{z}_h, \bar{z}_w, \bar{c}_v\}$, and y_B the mean of y in the benchmark: this is a utilitarian measure of gain (loss) in efficiency. Since our equilibrium is stationary, the square of the IGE measures the intergenerational component of the cross-sectional variance of lifetime earnings, while $\Delta \log \bar{c_v}$ measures the consumption equivalent welfare gain from a policy change in a utilitarian sense.

3.5 Discussion

While the concepts constructed above are useful for applying EOP definitions to quantitative economic models, many doors are left open. Perhaps the most significant shortcoming is the assumption of rationality: although we have categorized model states and outcomes as background, luck and effort, rationality implies that all decisions are indirect outcomes of their luck and effort. Hence, the model cannot differentiate between the several strands of "general- or factor-selective egalitarianism" discussed in Fleurbaey (1995), and we have taken a factor-selective approach by viewing conditional choices as effort ("will" in his paper). As such, his critique against factor-selective egalitarianism applies to our approach as well.

This is a feature of most standard quantitative economic models, and extracting those elements of individual choice that are entirely free of any influence from one's state is impossible with the assumption of rationality. While recent behavioural models may come closer, there does not yet exist a quantitative behavioural model suitable for analysing population distributions. Furthermore, the immense amount of data that would be needed to analyse such personal traits are not yet available either.[27]

Relatedly, our model is also restricted due to the assumption of identical preference parameters. We could hypothetically assume an artificial level of heterogeneity and also stochasticity in the subjective discount factor β, altruism factor θ, or risk aversion χ, the only

[27]For example, the General Social Survey, begun in 1972 and collected by the University of Chicago, includes many preference related questions that could potentially be decomposed into those influenced by circumstances or luck, or not. However, detailed economic information is limited. Relatedly, the German Socio-Economic Panel also includes some preference related questions, but too infrequently for such a decomposition.

preference parameters in our model. From an empirical point of view, we would again face the challenge that there does not exist any good data to discipline the level of heterogeneity/stochasticity in the population. Without such direct observation, if the model is asked to replicate the same set of moments but with different preferences, it would take away from the explanatory power of other elements of the model that generate heterogeneity.

As long as preferences are assumed to remain constant over an individual's lifetime, its quantitative effect would mainly be to explain outcomes that in its absence were explained by the intergenerational persistence of abilities, which we have categorized as a type (circumstance). And if we also categorize preferences as types, we expect it to play a similar role as the learning abilities in our model. For example, using a model with no ability persistence but an intergenerational 2-state Markov chain in which the discount factors β vary between high and low á la Krusell and Smith (1998), the qualitative results that follow remain unchanged.[28]

But as with luck and effort, there is a philosophical debate over how much of preferences should be considered circumstantial (Cohen, 1989). And by assuming identical preferences for all individuals, or that heterogeneous preferences as luck, we are implicitly assuming that assuming that individuals are not responsible for their preferences. Within this framework, we assert that the types and outcomes a planner considers when discerning EOP depend on who the planner views as being in control: the dynasty (in which case we would consider lifetime earnings conditional on parents' abilities) or an individual cut-off from dynastic links (in which case we would consider net wealth conditional on parents' lifetime earnings).

[28]Broadly speaking, the unobserved persistence of abilities is what fits the model to data moments that cannot be matched with the other parameters. So when we replace ability persistence with preference persistence, it plays a similar role. Of course we could arbitrarily choose the degree of preference heterogeneity and its persistence, but as long as moderate levels are chosen so as to not deviate too far from the data targets, the qualitative results remain unaltered.

4. Results

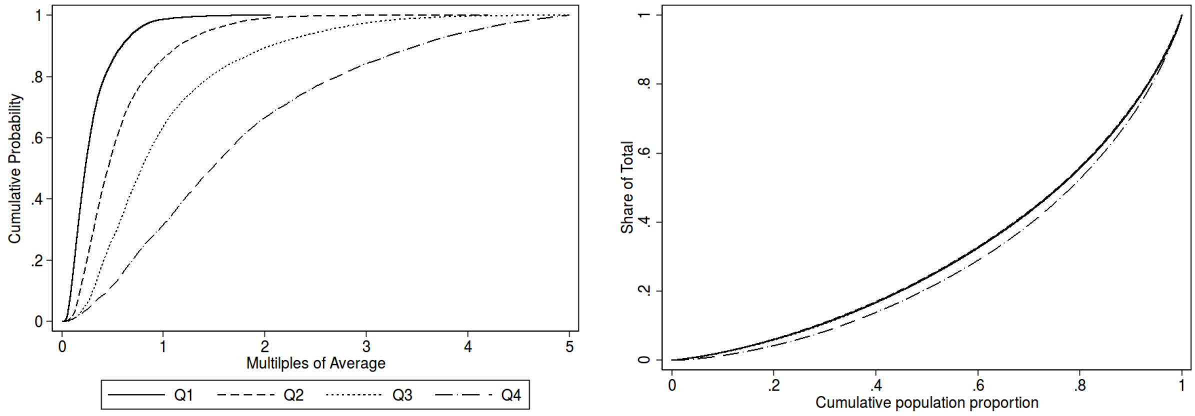
First, we analyse the benchmark equilibrium to demonstrate that accounting for intergenerational investments matter for EOP, both between and within types. We then turn to our counterfactual policy experiments.

4.1 Visual Inspection and Lorenz Dominance

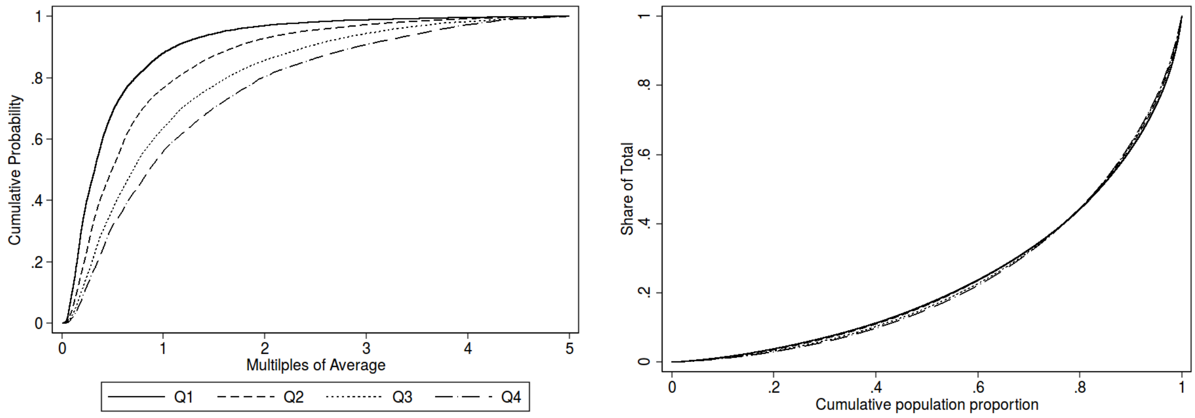
Figures 3 and 4 show the c.d.f.'s to the left and Lorenz curves to the right, for each of the cases 1.-5. Since the model is stylized, the equilibrium distribution is smooth and nicely shaped, unlike what is observed in the data. However, visual inspection reveals that our division of parental states, in all cases, qualitatively deliver an order of distributions that is comparable to what Lefranc *et al.* (2009) find for the empirical distributions in France, where they condition on parental occupations. What is visually clear, however, is that when we divide types by parental ability rather than lifetime earnings, the distance between the distributions shrink dramatically. It is also clear that when we condition on parental lifetime earnings but instead look at children's net wealth (z_w) as the outcome variable, the distance between the distributions widen.

Unfortunately, the smoothness of our simulated distributions give clear dominance orderings regardless of the visual distances—it is always the case that the conditional distributions of the higher parental quantiles first-order dominate the lower ones, regardless of how we divide types. In other words, the dominance orderings do not give us a sense of how large the levels of dominance is. This would not be the case in the real data that is observed with noise and includes more randomness than the channels we include in the model, or were we to assume some noise associated with observing the simulated variables. Instead of pursuing this route, below we instead turn to the Theil index for a cardinal comparison.

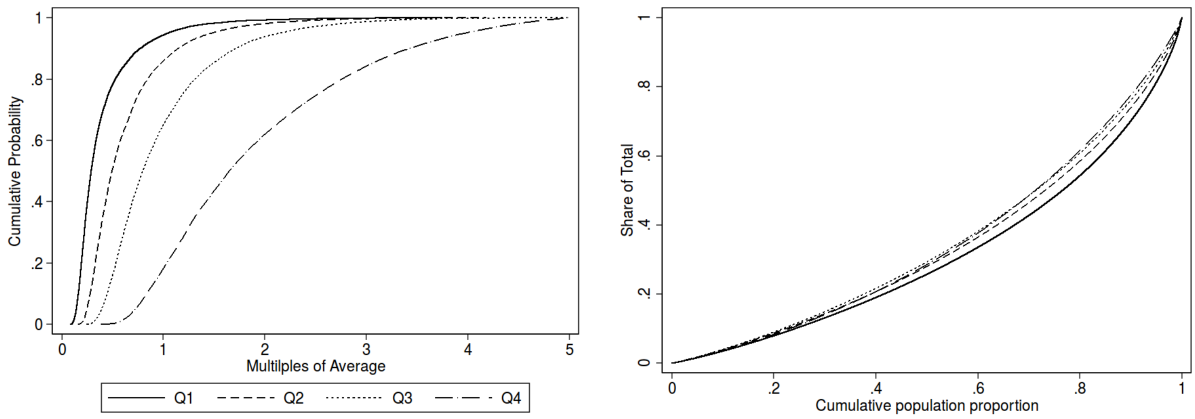
The Lorenz curves are also similar to Lefranc *et al.* (2009)'s findings, in the sense that



(a) Case 1: conditional distributions of z_h by $z_{h,-1}$ quartiles



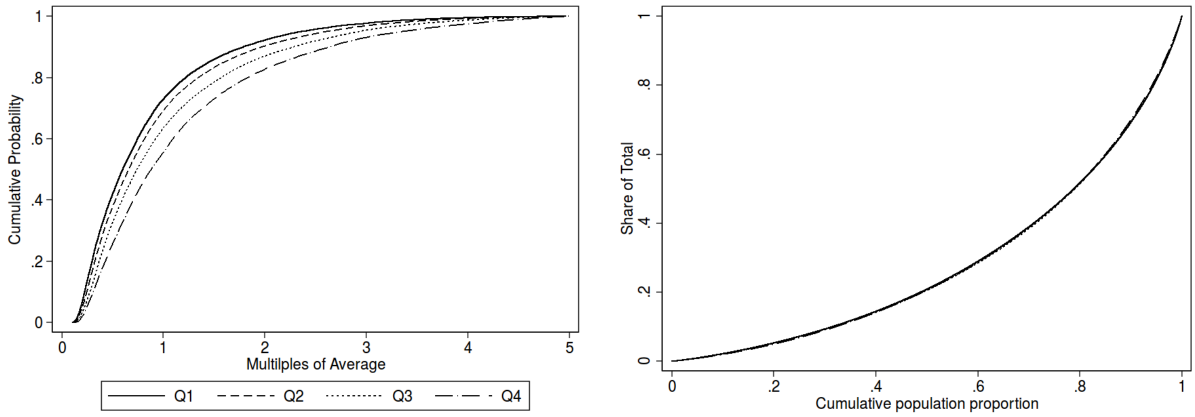
(b) Case 2: conditional distributions of z_h by a_{-1} quartiles



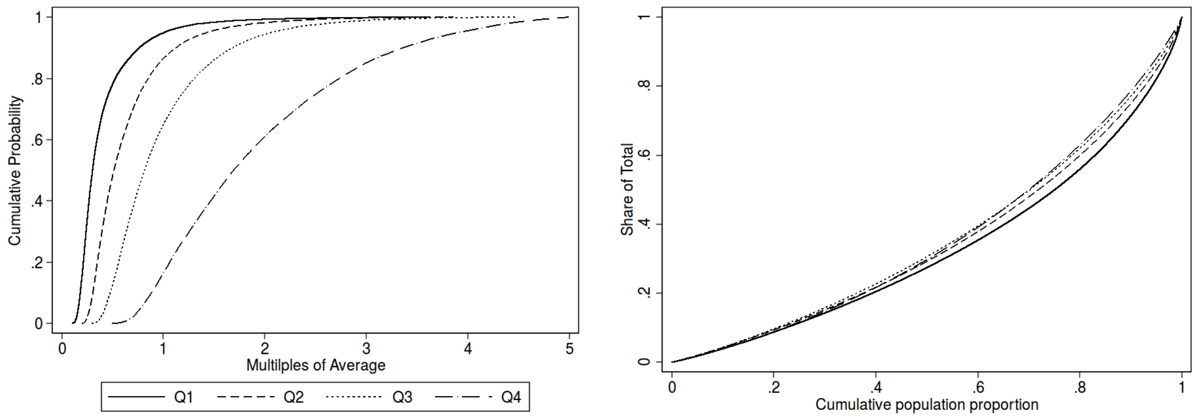
(c) Case 3: conditional distributions of z_w by $z_{h,-1}$ quartiles

Figure 3: Outcome distributions and Lorenz curves by type, cases 1-3

For each case, the variables are demeaned by the unconditional mean.



(a) Case 4: conditional distributions of c_v by a_{-1} quartiles



(b) Case 5: conditional distributions of c_v by $z_{h,-1}$ quartiles

Figure 4: Outcome distributions and Lorenz curves by type, cases 4-5
For each case, the variables are demeaned by the unconditional mean.

it appears as if at least visually, the outcome variables can be expressed as a fixed component that varies by type, multiplied by a random term that does not depend on type. This implies that relative luck and effort are rewarded similarly across types, once a first-order effect is accounted for, which is unexpected: Although we assume type-independent shocks, remember in fact that all the shocks are intercorrelated so that it is not clear why conditional outcomes should have similar spreads, not to mention the immense non-linearity associated with human capital acquisition. This is also confirmed in Table 6, where unlike the stochastic dominance tests, it is impossible to discern a clear pattern across the conditional distribution for most cases. What is even more striking is that again, when we condition on parental ability as depicted for cases 2. and 4., the overlap is even more extreme for both

Case 1. $(z_{h,-1}, z_h)$	Q2	Q3	Q4
Q1	=	=	?
Q2		>	?
Q3			?

Case 2. (a_{-1}, z_h)	Q2	Q3	Q4
Q1	>	>	<
Q2		?	?
Q3			<

Case 3. $(z_{h,-1}, z_w)$	Q2	Q3	Q4
Q1	<	<	<
Q2		<	?
Q3			?

Case 4. (a_{-1}, c_v)	Q2	Q3	Q4
Q1	=	=	<
Q2		=	<
Q3			<

Case 5. $(z_{h,-1}, c_v)$	Q2	Q3	Q4
Q1	<	<	<
Q2		<	?
Q3			?

Table 6: Lorenz Dominance tests

Case I, (x, y) refers to the each case I where x is the type variable and y is the outcome variable. Equality of the Lorenz curves is tested by Kolmogorov-Smirnov. Lorenz dominance is tested by testing 2nd order stochastic dominance of variables demeaned by type. We simulate p-values by re-centred bootstrap following Barrett and Donald (2003). The null is accepted if not rejected at the 95% confidence level, so all our notions are weak orderings.

$=$: Lorenz curves are equal.

$>_i$: row Lorenz dominates column.

$<_i$: column Lorenz dominates row.

?: cannot be ranked according to either test.

z_h and c_v, while the lowest type displays visually less equality when taking z_w or z_h as the outcome variable conditional on $z_{h,-1}$. For cases 3. and 5., the dominance relationships also exactly overlap.

In summary, we have preliminary confirmation that indeed, accounting for intergenerational investments comes closer to measuring the EOP of individual welfare than simply measuring the persistence of lifetime earnings across generations.

4.2 Theil Index Decomposition

The first column in Table 7 shows the Theil indices for our benchmark equilibrium, all normalized to lie between 0 and 100—hence, the quantitative values can be directly compared across all cases and groups.[29] The rows titled "Between" correspond to the between-type

[29] In Appendix Table E9, we show a similar decomposition of mean log deviations. The overall interpretation of the degree of EOP (or IOP) is similar to what we discuss here, based on Theil indices, but we find that the latter is more numerically robust.

indices, T_b. Our visual inspections from the previous subsection are numerically confirmed: notice that the between-type inequality comprises 46% of total inequality when conditioning on parental lifetime earnings, but drops to 8% when conditioning on parental abilities. Hence, if one takes the stance that intergenerational efforts should not be compensated away, the U.S. displays substantially higher EOP than might be expected. On the other hand, if we take the stance that intergenerational efforts *should* be compensated for, i.e., we define the child's outcome variable as z_w instead of z_h, EOP looks worse.

We argued in section 3 that, depending on which stance is taken on how types should be chosen, only one of the two comparisons should be made in light of intergenerational concerns. The Theil index offers a quantitative gauge of whether this is an appropriate measure by comparing the outcome distributions with the distributions of utility derived by the child generation. As seen in the first column of Table 7, the between-type Theil indices in cases 2. and 3. mirror cases 4. and 5., a confirmation that intergenerational concerns must be taken into account when measuring EOP—this is true regardless of whether one believes intergenerational efforts should be compensated for or not. The between index contribution for case 1. is 46%, while it is 52% for case 3.: looking at net wealth is more likely for us to conclude that EOP is less satisfied than lifetime earnings. Furthermore, this figure is similar to the 54% in case 5. Likewise, the between index contribution is a mere 8% in case 2., while in case 4. it is at its minimum, 2%. Hence EOP is almost satisfied if we believe that intergenerational efforts should be rewarded, and we get a similar picture if we looking at lifetime earnings directly as well (case 2).

But why is this the case? As discussed in Section 2.2, intergenerational persistence is strongly affected by human capital investments by the parent. Genetic abilities are not so much reflected across intergenerational earnings once early childhood investments are accounted for. Once this is understood, it is not so surprising that conditioning on parental

		B	E	G
	TOTAL	**4.32**	**3.52**	**4.56**
	Between	46%	43%	47%
	Within	54%	57%	53%
Case 1.	Q1	2.30	1.92	3.21
$(z_{h,-1}, z_h)$	Q2	2.22	2.00	2.22
	Q3	2.30	2.10	2.25
	Q4	2.94	2.51	2.92
	Between	8%	8%	8%
	Within	92%	92%	92%
Case 2.	Q1	4.59	3.90	4.90
(a_{-1}, z_h)	Q2	4.44	3.61	4.66
	Q3	4.44	3.59	4.76
	Q4	4.48	3.55	4.80
	TOTAL	**2.87**	**2.37**	**3.02**
	Between	52%	49%	53%
	Within	48%	51%	47%
Case 3.	Q1	2.38	1.99	2.59
$(z_{h,-1}, z_w)$	Q2	1.76	1.55	1.79
	Q3	1.45	1.32	1.45
	Q4	1.42	1.17	1.43

		B	E	G
	TOTAL	**2.66**	**2.14**	**2.79**
	Between	2%	3%	2%
	Within	98%	97%	98%
Case 4.	Q1	2.93	2.38	3.08
(a_{-1}, c_v)	Q2	2.95	2.36	3.10
	Q3	2.97	2.38	3.13
	Q4	2.91	2.30	3.08
	Between	54%	52%	56%
	Within	46%	48%	44%
Case 5.	Q1	2.11	1.90	2.31
$(z_{h,-1}, c_v)$	Q2	1.57	1.40	1.58
	Q3	1.28	1.19	1.28
	Q4	1.24	1.22	1.26

Table 7: Theil Index Decompositions

All indices are normalized to lie between 0 and 100. The rows "Between" and "Within" are the percentage contributions of the between- and within-type Theil indices to the total Theil index. Case $I.(x, y)$ refers to the each case I where x is the type variable and y is the outcome variable. B, E and G refer to different stationary equilibria: the benchmark, only education subsidies without lump-sum subsidies, and only lump-sum subsidies without education subsidies.

ability alone results in small differences in the lifetime earnings distributions of the child generation: the overlapping of the conditional distributions is an indicator that intergenerational human capital investments are being rewarded across multiple generations, and overwhelms any advantages that may come from having a high ability parent. This is confirmed by the fact that the between Theil contribution to the inequality between continuation values are minuscule when conditioning on parental ability. Perhaps even more surprisingly, the within-type Theil indices also tend to converge to each other, implying similar rewards to relatively similar levels of effort and luck.

In contrast, in cases 3. and 5. when we condition on parental lifetime earnings but instead look at z_w, the child's net wealth at middle age, as the outcome variable, EOP is even more strongly rejected than when looking at the child's lifetime earnings. This is again due to the importance of early childhood investments, but now interacting with intergenerational borrowing constraints. Children with low $z_{h,-1}$ parents have low z_h but even lower z_w because of the binding constraints. The incentive to invest in children is large and if possible the current child generation would want to borrow from the next (grandchild) generation, but they are not permitted to do so and hit the zero bequest constraint. This is the Becker-Tomes mechanism: parents who are not able to invest optimally in their children are even more constrained financially. That this is the proper outcome variable for which EOP should be considered is again confirmed by the fact that the between Theil contribution to the inequality between continuation values mirrors closely that of the inequality between z_w rather than z_h.

The coincidence of the Lorenz curves in cases 2. and 4., and widening gaps between the curves in cases 3. and 5. is also confirmed by comparison of the within Theil indices. These are remarkably close to each other in the former case, reinforcing the strong dependence on early human capital investments of the model. In contrast, in cases 3. and 5., it is the lower types, and especially the lowest type, that has the least similar index. Given that lower types will have the more intergenerationally borrowing constrained parents, not only between all types but even within the lower types, inequality of opportunities are magnified.

4.3 Policy Analysis

Given our benchmark parametrization, we compare two different *stationary equilibria*: shifting all lump-sum subsidies to education subsidies, and shifting all education subsidies to lump-sum subsidies. In the tables that follow, we label these cases as E (only education subsidies) and G (only lump-sum subsidies), respectively, and B refers to the benchmark. We also summarize the *short-run* outcomes in partial equilibrium in Appendix F.2, rather than in long-run stationary equilibrium. In this case, we keep the level of other policies at their

	B	E	G
IGE	0.60	0.53	0.60
Gini z_h	0.48	0.44	0.49
RW Gini	0.62	0.59	0.63
$\Delta \log \bar{z_h}$	-	34%	-1%
$\Delta \log \bar{z_w}$	-	29%	-1%
$\Delta \log \bar{c_v}$	-	28%	-1%

Table 8: Counterfactual moments (IGE and Gini's) and Average Outcomes
IGE is the correlation of lifetime earnings across generations. z_h is lifetime earnings, and (z_w, c_v) are defined in the text. RW stands for "Retirement Wealth." For each y, $\bar{y}$ refers to its cross-sectional average. B, E and G refer to different stationary equilibria: the benchmark, only education subsidies without lump-sum subsidies, and only lump-sum subsidies without education subsidies.

benchmark values (that is, taxation and social security policies are held constant and not adjusted to satisfy the government budget balance condition).

We focus on long-run equilibria since policies that seek to improve intergenerational outcomes are focusing on long-run outcomes by nature: that is, a policy implemented to improve children's outcomes is inherently targeting adult outcomes a generation later. Both in the real-world and in the data, the effect of such policies would not be expected to manifest themselves immediately. Indeed in Appendix F.2, we show that the short-run impact of the policy changes are negligible compared to their long-run effects.

Both policies are inherently redistributive: education subsidies support the human capital formation of low income families, and lump-sum subsidies provide public insurance in the face of idiosyncratic risk, which is mostly important for low-income families. By design of holding the total amount of subsidies fixed, they may also crowd out each other: education subsidies may lead to too much education at the expense of other consumption-saving opportunities, while lump-sum subsidies can discourage human capital investment and reduce long-run per capita income or welfare.[30] To attain a sense of the effect that such changes will have on equity and efficiency, we first present the IGE, average outcomes and

[30] Of course there can be transition costs associated with increasing human capital when education subsidies are too high as well, but our goal is to demonstrate a divergence between equality of outcomes and EOP, not compute exact welfare gains or losses as in the conventional economics literature (e.g. Krueger and Ludwig (2013); Badel and Huggett (2014)).

Gini indices of lifetime earnings and retirement wealth for each case. While the change in magnitudes, especially for education subsidies, seem high, a comparison of the Theil indices reveal that EOP is barely affected.

Table 8 shows that education subsidies reduce both the IGE and Gini's, consistent with the "Gatsby" curve found in empirical work (Corak, 2013). This is because they directly affect human capital investments by ensuring a certain level of human capital. The impact of education subsidies must be analysed in two directions: column E represents an increase, and G a reduction, while keeping the overall level of subsidies neutral relative to average earnings. Observe that even when education subsidies are compensated for with additional lump-sum subsidies (column G), it leads to an increase in inequality of outcomes, albeit small. The two columns together imply that enough insurance is being provided by the lump-sum subsidies, so that it is better to incentivise a higher level of education which leads to more equal outcomes.

Not only do education subsidies equalize outcomes, it is also substantively efficient—all variables (z_h, z_w, c_v) increase by significant amounts, in particular, consumption equivalent welfare increases by as much as 28%. While the magnitude is somewhat large, we are not computing transition costs nor modelling exactly how the subsidies are financed. In any case, the fact that education subsidies can enhance both equity and efficiency itself is not so surprising, especially in light of the dynamic complementaries we model for human capital acquisition (Benabou, 1996; Cunha and Heckman, 2007).

What *is* surprising is that, looking back at Table 7, EOP as measured by the contribution of the between-type Theil index toward the total Theil index barely changes compared to the benchmark. This is not because the Theil index measures something orthogonal to the Gini index: the total Theil decreases significantly with more education subsidies (column E). Despite this fact, for cases 2. and 4., there seems to be no change in EOP compared to the benchmark at all. This is for two reasons: for cases 2. and 4., EOP is strongly present to begin with, and the relatively small size of the education subsidies (approximately 5% of average income) does little to push it down further. Furthermore, as should be clear by now our outcome variables are highly dependent on how much human capital each individual accumulates and even more importantly, how much they invest in their children. Given that cases 2. and 4. measure EOP under the view that intergenerational efforts should be rewarded, education subsidies, which favour the offspring of those dynasties for whose ancestors exerted *less* effort, should not be expected to improve.

But then what about cases 3 and 5? First note that while the changes are small, there still is a positive improvement in EOP (as measured by the "Between" row). Closer inspection of the within-type Theil indices reveal that this is associated with the lowest type "catching up" with the other types, in terms of rewarding relative luck and efforts equally. Recall that the lower types are the most intergenerationally constrained, so there within exists the largest room for improvements in EOP. The fact that education subsidies reduce this gap indicate that the subsidies are alleviating the intergenerational borrowing constraints, but that the magnitudes are not enough. So for both between and within types, the size of the subsidies must be much larger, and/or designed to be much more progressive, to achieve higher levels of EOP.

5. Conclusion

In this paper, we apply EOP concepts to a quantitative economic model featuring a rich mechanism of human capital investment, intergenerational borrowing constraints, and government policies. We emphasize that, rather than looking at the lifetime earnings across two generations, intergenerational investments should be accounted for before analysing the extent to which EOP is present. In comparison to EOP implications when only considering lifetime earnings, we find little evidence for lack of EOP if intergenerational investments should be rewarded, while it is even more lacking if such investments should not be rewarded. We propose which variables should be considered to arrive at these conclusions and validate our proposals by also considering the distributions of continuation values. We also showed that, focusing all subsidies on education can increase efficiency while reducing inequality, but may still be not enough to rectify EOP at empirically observed levels of subsidies.

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Appendices

A. Period-to-Period Formulation of Decision Problems

For any age j, it is useful to define the state vector $\mathbf{x}_j = [S, a, \epsilon; h_j]$: educational attainment (college or not, $S \in \{1, 0\}$), ability, market luck shock, and current human capital level of an individual in period j of life. Let $(y, o) = (4, 8)$, i.e., the first periods in the young parent and grandparent stages, respectively.

Young parent ($y = 4$) Note that there are no realizations of shocks between $j = 4$ to 7, which effectively collapses the college choice into the young parent's problem. Symmetrically, there are no realizations of shocks for the grandparent throughout the same period. The timing of the model is such that a young parent anticipates the bequests b that will be decided by his now old parent (the grandparent) when he becomes 42, or $j = 7$. Hence, we are assuming that first, the grandparent makes a bequest decision b, followed by a joint decision between the young parent and child on whether the child should go to college, $S' \in \{0, 1\}$.

Suppose the family chooses education level S' for the child. The young parent recognizes that any earnings made in the previous period and today ($j = 3, 4$) will not matter for social security. Hence his state is his own state vector $\mathbf{x}_y$, the anticipated bequests b, and the ability of his child, born in the beginning of this period, a'. Given this state, he chooses consumption streams for himself and his child, $c = \{c_{y+l}, c'_{y+l-4}\}_{l=0}^{2}$, own human capital investments and savings $(n, s) = \{n_{y+l}, s_{y+l+1}\}_{l=0}^{2}$, and investments in his child's human capital formation, (n_p, m_p). Furthermore, the family jointly decides whether the child should attend college at $j = 7$, and how much the child should contribute toward joint family earnings versus the child's human capital accumulation at age $j' = 3$. The value function is

$$
\begin{aligned}
& W_y(S', a'; \mathbf{x}_y; b) \\
&= \max_{c,s,n,n_p,m_p} \left\{ \sum_{l=0}^{3} \beta^l \left[u(c_{y+l}) + \theta u(c'_{y+l-4}) \right] + \beta^4 \int_{a'',\epsilon'} V_o(a''; \mathbf{x}'_y; \mathbf{x}_o, \bar{e}_o; s_o + b) dF(a'', \epsilon' | a') \right\}
\end{aligned}
$$

subject to consumption-savings decisions

$$
\begin{aligned}
& c_{y+l} + c'_{y+l-4} + s_{y+l+1} = f(e_{y+l}) + (1+\tilde{r}) s_{y+l} + q_A \cdot g, \qquad \text{for } l = 0, 1, \\
& c_{y+2} + c'_{y-2} + s_{y+2+1} + m_p = f(e_{y+2}) + (1+\tilde{r}) s_{y+2} + q_A \cdot g, \\
& c_{y+3} + c'_{y-1} + s_{y+4} = f(e_{y+3}) + (1+\tilde{r}) s_{y+3} + f(e'_{y-1}) - S' \cdot \kappa + 2g
\end{aligned}
$$

and human capital accumulation decisions

$$
\begin{aligned}
& e_y = w_S h_y \epsilon (1 - n_y - n_p) \quad \text{and} \quad n_y \in [0,1], \quad n_p \in [0, n_y], \\
& e_{y+l} = w_S h_{y+l} \epsilon (1 - n_{y+l}) \quad \text{and} \quad n_{y+l} \in [0,1], \qquad \text{for } l = 1,2,3, \\
& e'_{y-1} = w_{S'} h'_{y-1} (1 - n'_{y-1}) \quad \text{and} \quad n'_{y-1} \in [0,1], \\
& \bar{e}_o = \frac{e_{y+1} + e_{y+2} + e_{y+3}}{3}.
\end{aligned}
$$

where his own and child's human capital production, and earnings net of taxes $f(\cdot)$, are specified in Sections 1.2-1.3. The transfer g is multiplied by an adult equivalence scale q_A when the child is young, while a college aged child ($j' = 3$) receives the full subsidy. The parent and child are effectively solving a Pareto problem, where the child has a Pareto weight of $\frac{\theta}{1+\theta}$, as determined by the parent's altruism.[31]

[31] Additionally, we could also assume that the parent makes college transfers at $j = 3$, and the child saves for independence, s_{y-1}, but the timing of the problem would imply both are zero.

Given this, the household chooses S':

$$V_y(a'; \mathbf{x}_y; b) = \max_{S' \in \{0,1\}} \left\{ W_y(S', a'; \mathbf{x}_y; b) \right\}.$$

We assume a fixed cost κ for college, as our focus is more on parental influences on earlier stages of education rather than college; furthermore it turns out the college margin does not play a quantitatively large role, as discussed in the text.

Grandparent ($o = 8$) From the young parent's problem, it is apparent that the grandparent's states are the ability of his grandchild a'', the state of the young parent, $\mathbf{x}'_y$, and his own state $(\mathbf{x}_o, s_o)$ and $(\bar{e}_o, b)$. In particular, b is the bequest his retired parent (the great-grandparent) decided when the grandparent was still a young parent, but only received when old. Given this, he makes consumption, own human capital investment, and savings decisions, $(c, s, n) = \{c_{o+l}, n_{o+l}, s_{o+l+1}\}_{l=0}^{4}$. In period $j + 3 = 11$ he leaves bequests, b', for the young parent, which is received in the next period. His value function is

$$V_o(a''; \mathbf{x}'_y; \mathbf{x}_o, \bar{e}_o; s_o, b) = \max_{c,n,s,b'} \left\{ \sum_{l=0}^{4} \beta^l u(c_{o+l}) + \theta V_y(a''; \mathbf{x}'_y; b') \right\}$$

where maximization is subject to

$$c_o + s_{o+1} = f(e_o) + (1 + \tilde{r})(s_o + b) + g,$$

$$c_{o+l} + s_{o+l+1} = f(e_{o+l}) + (1 + \tilde{r})s_{o+l} + g, \qquad \text{for } l = 1, 2,$$

$$c_{o+3} + s_{o+4} + b' = p(\bar{e}) + (1 + \tilde{r})s_{o+3} + g, \qquad b' \geq 0,$$

$$c_{o+4} + s_{o+5} = p(\bar{e}) + (1 + \tilde{r})s_{o+4} + g, \qquad s_{o+5} \geq 0$$

$$e_{o+l} = w_S h_{o+l} \epsilon (1 - n_{o+l}) \quad \text{and} \quad n_{o+l} \in [0, 1], \qquad \text{for } l = 0, 1, 2,$$

$$\bar{e} = \bar{e}_o / 2 + \sum_{l=0}^{2} e_{o+l} \Big/ 6.$$

The parent faces only a natural borrowing constraint on own savings ($s_{o+5} \geq 0$), but bequests must be non-negative ($b' \geq 0$); i.e the parent cannot borrow against their child's income. This is in the spirit of Loury (1981), where parents cannot borrow against their children's future income to invest in their children's human capital.

Again, notice the timing of the bequests: we assume that the bequests are decided upon *after* the grandchild's college decisions S'', but *before* the realization of (a''', ϵ''), i.e. the great-grandchild's ability shock and grandchild's market luck shock. However, the transfer *occurs* after their realizations. This keeps the problem convex, and given the time horizon, does not seem to be an unreasonable assumption.

B. Simplifying the Household's problem

Our assumptions allow the reduction of choice variables and states as follows.

B.1 Consumption Smoothing problem

Denote the young parent household's total consumption in period j as $C_j = c_j + c'_{j-4}, j = 4, \ldots, 7$. Since the young household solves a Pareto problem, we have

$$c_j = C_j \Big/ \left(1 + \theta^{1/\chi}\right), \qquad c'_{j-4} = \left(\theta^{1/\chi} C_j\right) \Big/ \left(1 + \theta^{\frac{1}{\chi}}\right)$$

which allows us to define a household utility function

$$U(C_j) = qu(C_j), \qquad q = \left(1 + \theta^{\frac{1}{\chi}}\right)^{\chi}.$$

Similarly, consumption is smoothed in all periods where there is no realization of a new shock, namely throughout the young household period $j = 4, 5, 6, 7$ and old adult period

$j = 8, \ldots, 12$. For both stages we can define a utility function in terms of the PDV of total consumption within each stage:

$$
\begin{aligned}
U_y(C_y) =& q_y u(C_y), \quad q_y = q \left[\sum_{j=4}^{7} \left(\beta^{\frac{1}{\chi}} (1+\tilde{r})^{\frac{1}{\chi}-1} \right)^{j-4} \right]^{\chi}, \quad C_y = \sum_{j=4}^{7} \frac{C_j}{(1+\tilde{r})^{j-4}} = \left(\frac{q_y}{q} \right)^{\frac{1}{\chi}} C_4 \\
U_o(C_o) =& q_o u(C_o), \quad q_o = \left[\sum_{j=8}^{12} \left(\beta^{\frac{1}{\chi}} (1+\tilde{r})^{\frac{1}{\chi}-1} \right)^{j-8} \right]^{\chi}, \quad C_o = \sum_{j=8}^{12} \frac{c_j}{(1+\tilde{r})^{j-8}} = q_o^{\frac{1}{\chi}} c_8.
\end{aligned}
$$

B.2 Lifetime Income Maximization problem

Given the timing of shocks and choices, the parent's own human capital decision problem is deterministic and can be solved as a lifetime income maximization problem at age $y+1=5$. Given $\mathbf{x}_{y+1}$, the PDV of lifetime earnings is

$$
z_{h,y+1}(\mathbf{x}_{y+1}) = \max_{\{n_{y+l}\}_{l=1}^{6}} \left\{ \sum_{l=1}^{6} \frac{1}{(1+\tilde{r})^{l-1}} \cdot \left[1 - \tau_e(e_{y+l}) - \tau_s + \frac{(2+\tilde{r})p_1}{6(1+\tilde{r})^{8-l}} \right] e_{y+l} \right\}, \quad \text{(B.1)}
$$

where

$$
e_{y+l} = w_S h_{y+l} \epsilon (1 - n_{y+l}), \qquad h_{y+l+1} = a(n_{y+l} h_{y+l})^{\gamma_S} + h_{y+l}.
$$

This problem can be easily solved recursively. Let

$$
\tilde{y}_{y+l} = \left\{ 1 - \tau_e(e_{y+l}) - \tau_s + (2+\tilde{r})p_1 \Big/ \left[6(1+\tilde{r})^{8-l} \right] \right\} e_{y+l},
$$

then given $n_{10} = 0, z_{h,10} = \tilde{y}_{10}$,

$$
z_{h,j}(\mathbf{x}_j) = \max_{n_j} \left\{ \tilde{y}_j + z_{h,j+1}(\mathbf{x}_{j+1}) \Big/ (1+\tilde{r}) \right\}
$$

for $j = 5, \cdots, 9$. Hence the young parent can take $z_{h,y+1}$ as given.

B.3 Reformulation of the value functions

We now reformulate the young and old's value functions. The grandparent of stage $o = 8$ solves

$$V_o(a''; S', a', \epsilon', h'_y; z_o) = \max_{C_o, z'_y} \left\{ q_o u(C_o) + \theta V_y(a''; S', a', \epsilon', h'_y; z'_y) \right\}$$

$$C_o + z'_y = z_o, \qquad z'_y \geq 0,$$

where the new state z_o summarizes the lifetime PDV of savings and earnings carried over from when young, and $b' = (1 + \tilde{r})^3 z'_y$ are the implied bequests. The young household of stage $y = 4$ decides the child's skill level

$$V_y(a'; S, a, \epsilon, h_y; z_y) = \max_{S' \in \{0,1\}} \left\{ W_y(S', a'; S, a, \epsilon, h_y; z_y) \right\},$$

where

$$W_y(S', a'; S, a, \epsilon, h_y; z_y) = \max_{C_y, z, n_y, n_p, m_p, n_k} \left\{ q_y u(C_y) + \beta^4 \int_{a'', \epsilon'} V_o(a''; S', a', \epsilon', h'_y; z_o) dF(a'', \epsilon' | a') \right\}$$

subject to

$$C_y + \frac{z_o}{(1 + \tilde{r})^4} + \frac{m_p}{(1 + \tilde{r})^2} = f(e_y) + \frac{z_{h,y+1}(S, a, \epsilon, h_{y+1})}{1 + \tilde{r}} + \frac{f(e_k) - S' \cdot \kappa}{(1 + \tilde{r})^3} + z_y + G,$$

$$e_y = w_S h_y \epsilon (1 - n_y - n_p) \quad \text{and} \quad n_y, n_p \geq 0, \ n_y + n_p \leq 1,$$

$$e_k = w_{S'} h_k (1 - n_k) \quad \text{and} \quad n_k \in \left[2S'/3, 1 \right],$$

$$h_{y+1} = a(n_y h_y)^{\gamma_S} + h_y, \qquad h'_y = a' n_k^{\gamma_{S'}} h_k^{\gamma_p},$$

$$h_k = \left[\gamma_k^{\frac{1}{\phi}} (m_p + d)^{\frac{\phi - 1}{\phi}} + (1 - \gamma_k)^{\frac{1}{\phi}} \left(n_p h_y \right)^{\frac{\phi - 1}{\phi}} \right]^{\frac{\phi}{\phi - 1}},$$

and (e_k, h_k, n_k) denote, respectively, the child's earnings, human capital, and time spent in college (period 3). The last term in the budget constraint G is

$$G = g \cdot \left[\sum_{l=y}^{y+2} \frac{A}{(1+\tilde{r})^{l-y}} + \frac{2}{(1+\tilde{r})^3} + \sum_{l=o}^{o+4} \frac{1}{(1+\tilde{r})^{l-y}} \right] + \frac{p_0(2+\tilde{r})}{(1+\tilde{r})^8},$$

is the lifetime PDV of subsidies.

C. Solving the Household's problem

C.1 Grandparent's problem

When solving the grandparent's problem, notice that V_y will have kinks according to the grandchild's college decision. However, we know exactly where those kinks are, i.e. we know the $\hat{z}_y(a''; \mathbf{x}'_y) \geq 0$ s.t.

$$W_y(0, a''; \mathbf{x}'_y; \hat{z}_y(a''; \mathbf{x}'_y)) = W_y(1, a''; \mathbf{x}'_y; \hat{z}_y(a''; \mathbf{x}'_y)),$$

so we can search for the solution before and after the kink and choose the max. The necessary condition for the grandparent's optimization problem is

$$q_o u'(C_o) \geq \theta V_{y,6}(a''; S', a', \epsilon', h'_y; z'_y), \qquad \text{with equality if } z'_y > 0, \tag{C.1}$$

and by the envelope theorem

$$V_{o,6}(a''; S', a', \epsilon', h'_y; z_o) = q_o u'(C_o).$$

The simplification also makes clear why parents who leave bequests are not necessarily those who were not able to invest efficiently in their children. In fact, inspection of the grandparent's simplified problem reveals that all else equal, parents with high ability and/or human capital children will leave *less* bequests (as long as the value function is monotone).

C.2 Young parent's problem

Note that due to standard Inada conditions, in an optimum $[z_o, n_y, n_p, n_k] > 0$. The possible corner solutions are when $n_y + n_p = 1$ and/or n'_k hitting either boundary; and/or $m_p = 0$, which happens when the subsidy d is larger than the private optimum.

Euler Equations The optimality condition for z_o implies

$$q_y u'(C_y) = [\beta(1+\tilde{r})]^4 J_4 \tag{C.2}$$

where

$$J(S', a', h'_y; z_o) = \int_{a'', \epsilon'} V_o(a''; S', a', \epsilon', h'_y; z_o) dF(a'', \epsilon' | a'),$$

while the envelope theorem implies

$$W_{y,7}(S', a'; S, a, \epsilon, h_y; z_y) = q_y u'(C_y).$$

The first order conditions w.r.t. n_k in an interior are

$$q_y u'(C_y) \cdot y'(e_k) w_{S'} h_k = \beta^4 (1+\tilde{r})^3 J_3 \cdot \gamma_{S'} h'_y \Big/ n_k.$$

It may well be the case that $n_k = 1$, or $2/3$ for college kids, in which case the equality does not hold.

Human Capital Formation The parent is forward-looking and takes the child's choices into consideration. The optimality conditions when $n_y + n_p < 1$ are (note that $n_y, n_p > 0$ strictly due to Inada conditions):

$$
\begin{aligned}
n_y: &\quad y'(e_y) w_S \epsilon = \frac{z_{h,y+1,4}}{1+\tilde{r}} \cdot \frac{\gamma_S a (n_y h_y)^{\gamma_S}}{n_y h_y}, \\
n_p: &\quad q_y u'(C_y) \cdot y'(e_y) w_S \epsilon = \beta^4 J_3 \cdot \frac{\gamma_p h'_y}{h_k} \cdot \left[\frac{(1-\gamma_k) h_k}{n_p h_y} \right]^{\frac{1}{\phi}}, \\
m_p: &\quad q_y u'(C_y) \geq \beta^4 (1+\tilde{r})^2 J_3 \cdot \frac{\gamma_p h'_y}{h_k} \cdot \left(\frac{\gamma_k h_k}{m_p + d} \right)^{\frac{1}{\phi}}, \qquad \text{with equality if } m_p > 0.
\end{aligned}
\tag{C.3}
$$

If $n_y + n_p = 1$, the parent divides time between himself and his child so that the returns are equal:

$$
q_y u'(C_y) \cdot \frac{z_{h,y+1,4}}{1+\tilde{r}} \cdot \frac{\gamma_S a (n_y h_y)^{\gamma_S}}{n_y h_y} = \beta^4 J_3 \cdot \frac{\gamma_p h'_y}{h_k} \cdot \left[\frac{(1-\gamma_k) h_k}{n_p h_y} \right]^{\frac{1}{\phi}}. \tag{C.4}
$$

The choice for (m_p, n_k) can be simplified as:

$$
z_{h,y+1,4} \cdot \frac{\gamma_S a (n_y h_y)^{\gamma_S}}{n_y h_y} = \begin{cases} \frac{1}{1+\tilde{r}} \left[\frac{1-\gamma_k}{\gamma_k} \cdot \frac{m_p + d}{n_p h_y} \right]^{\frac{1}{\phi}} & \text{if } m_p \geq 0, \\ \frac{\gamma_p}{\gamma_{S'}} \cdot \frac{y'(e_k) w_{S'} n_k}{(1+\tilde{r})^2} \cdot \left[\frac{(1-\gamma_k) h_k}{n_p h_y} \right]^{\frac{1}{\phi}} & \text{if } n'_k \leq 1. \end{cases} \tag{C.5}
$$

Basically, all this says is that in an interior solution, the optimal solution for m_p is such that the time-goods investment ratio in the child's human capital formation is a power of the costs; and the college-age student's time cost per unit of stage y human capital must equal the parent's.

D. Solution Method

Our SMM is a nested fixed point problem. For each Θ, we obtain individual decision rules, the stationary distribution, and find $(\beta, \mu, \bar{\mathbf{e}})$ that solve (4)-(5). For given Θ, we compute:

1. Guess $(\beta, \mu, \bar{\mathbf{e}})$.

2. Obtain individual choices.

   (a) Outside value function iteration, solve (B.1). For each state, store all values of $\{h_j, n_j\}_{j=5}^{10}$. Obtain $z_{h,y+1}, z_{h,y+1,4}$.

   (b) Guess J, J_4, and solve for $W_y, W_{y,7}$ as follows. In the most outer-loop, solve for (n_y, n_p) using Brent's method:

      i. Assuming $n < 1$, solve for n_p. Note that n_y can be solved for without the value function by (C.3).

      ii. Optimize over n_p by (C.4)—(C.5) give closed form solutions for (m_p, n_k).

      iii. Solve for z_o using the Euler Equation (C.2).

      iv. Now resolve the problem assuming $n_y + n_p = 1$ ($n_y = 1 - n_p$), and solve for n_p. Check whether or not $n = 1$ by comparing values from each case.

   (c) Obtain $\hat{z}_y(a'; \mathbf{x}_j)$: interpolate to find where $W_y(0, a'; \mathbf{x}_j; \hat{z}_y) = W_y(1, a'; \mathbf{x}_j; \hat{z}_y)$.

   (d) Solve grandparent's problem (choose z'_y) using Euler Equation (C.1).

   (e) Integrate over Store $V_o, V_{o,6}$ w.r.t. (a'', ϵ') to obtain J', J'_4.

   (f) Repeat 2(b) until $(J', J'_4) \approx (J, J_4)$.

3. Due to the size of the state space, we obtain the stationary distribution via Monte-Carlo simulation rather than directly approximating the distribution.

   (a) Simulate $N = 120,000$ households for $T = 200$ generations, using decision rules obtained above.

(b) Obtain implied life-cycle values of $\{n_j, h_j\}_{j=4}^{10}$ from 2(a). From this, we can compute each individual's consumption, savings and earnings decisions.

(c) Aggregate to obtain implied $(r, EP, \bar{\mathbf{e}}')$.

4. Iterate from 1 until $(r, EP, \bar{\mathbf{e}}) \approx (4\%, 46.8\%, \bar{\mathbf{e}}')$.

For each Θ, we solve (10) with equal weights on all moments. In each iteration, the equilibrium vector $(\beta, \mu, \bar{\mathbf{e}})$ is solved for by numerical iteration. The choice of (N, T) is arbitrary, but increasing to $N = 240,000$, $T = 300$ had no effects. Further details are available upon request.

E. Mean Log Deviations

In the text, we used the Theil index as our measure of IOP. Following Ferreira and Gignoux (2011), we show the same results using mean logarithm deviations, defined as

$$\begin{aligned} MLD &= \sum_{j=1}^{J} \frac{\mu_j}{\mu} \cdot \int \log \frac{y}{\bar{y}} dF_j(y) = \sum_{j=1}^{J} s_j \left[\log \frac{\bar{y}_j}{\bar{y}} + \int \log \frac{y}{\bar{y}_j} dF_j(y) \right] \\ &= MLD_b + \sum_{j=1}^{4} s_j MLD_j \end{aligned}$$

where the variable definitions are the same as in Section 3.4. The results are summarized in Table E9 and are qualitatively in line with our results using the Theil index in Table 7. Numerically, we find that the Theil index is more stable when the simulated data contains many extreme points (close to zero). In particular, this is what makes the relationship between cases 2 and 4 tighter when using the Theil index rather than the MLD.

		B	E	G
	TOTAL	**0.53**	**0.43**	**0.52**
	Between	49%	45%	55%
	Within	51%	55%	45%
Case 1. $(z_{h,-1}, z_h)$	Q1	0.25	0.20	0.18
	Q2	0.24	0.21	0.24
	Q3	0.25	0.23	0.25
	Q4	0.35	0.29	0.34
	Between	14%	17%	14%
	Within	86%	83%	86%
Case 2. (a_{-1}, z_h)	Q1	0.46	0.38	0.45
	Q2	0.47	0.37	0.45
	Q3	0.49	0.39	0.49
	Q4	0.51	0.39	0.50
	TOTAL	**0.35**	**0.29**	**0.37**
	Between	51%	48%	53%
	Within	49%	52%	47%
Case 3. $(z_{h,-1}, z_w)$	Q1	0.22	0.19	0.18
	Q2	0.17	0.15	0.24
	Q3	0.14	0.13	0.25
	Q4	0.15	0.12	0.34

		B	E	G
	TOTAL	**0.32**	**0.26**	**0.35**
	Between	0%	1%	0%
	Within	100%	99%	100%
Case 4. (a_{-1}, c_v)	Q1	0.31	0.25	0.33
	Q2	0.31	0.25	0.34
	Q3	0.31	0.26	0.34
	Q4	0.32	0.25	0.35
	Between	54%	51%	55%
	Within	46%	49%	45%
Case 5. $(z_{h,-1}, c_v)$	Q1	0.19	0.16	0.22
	Q2	0.15	0.13	0.15
	Q3	0.13	0.11	0.12
	Q4	0.13	0.10	0.13

Table E9: Mean Log Deviations

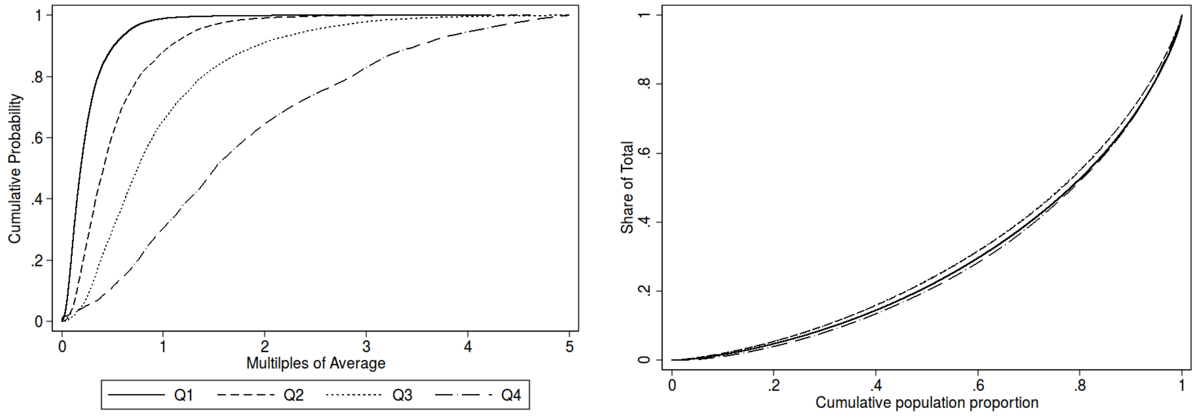
The rows "between" and "Within" are the percentage contributions of the between- and within-type MLDs to the total MLD. Case $I.(x, y)$ refers to the each case I where x is the type variable and y is the outcome variable. B, E and G refer to different stationary equilibria: the benchmark, only education subsidies without lump-sum subsidies, and only lump-sum subsidies without education subsidies.

F. Additional EOP Analysis

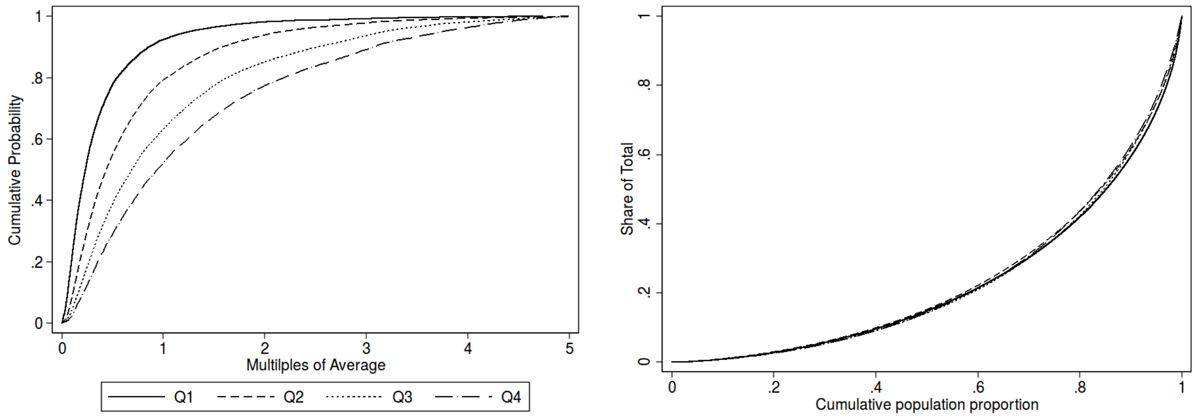
F.1 No correlation between own abilities and luck ($\rho_\epsilon = 0$)

In Section 2.2, we showed the effect of setting $\rho_\epsilon = 0$ on a select number of moments. In this appendix, we display our ordinal and cardinal EOP measures under this assumption (in the long-run stationary equilibrium). Figures F5-F6 shows the type-contingent outcomes, and Table F10 the Theil index and MLD decomposition. Since we already know that the smoothness of the simulated distributions makes ordinal comparisons difficult, we omit results from the Lorenz dominance tests.

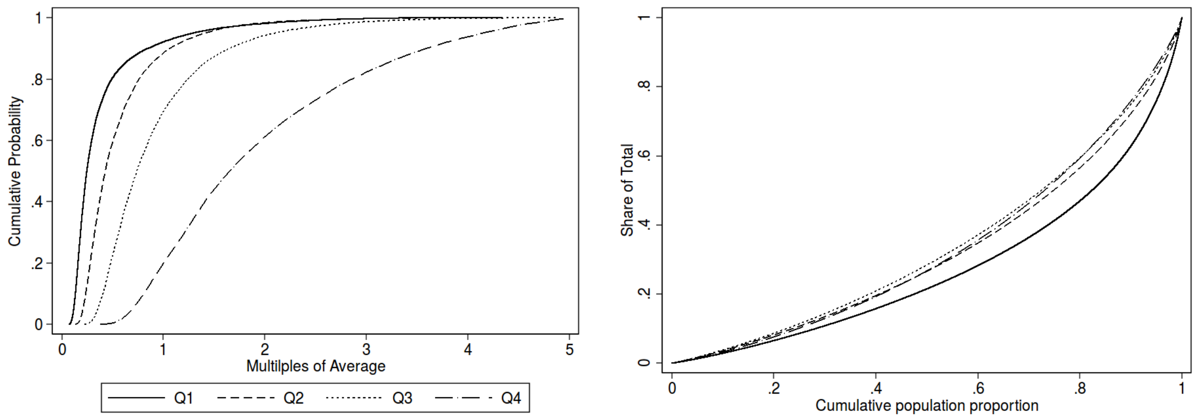
The overall message that EOP is larger when types are divided by parental ability rather than parents' income, and that EOP is smaller when looking at children's net wealth rather



(a) Case 1: conditional distributions of z_h by $z_{h,-1}$ quartiles



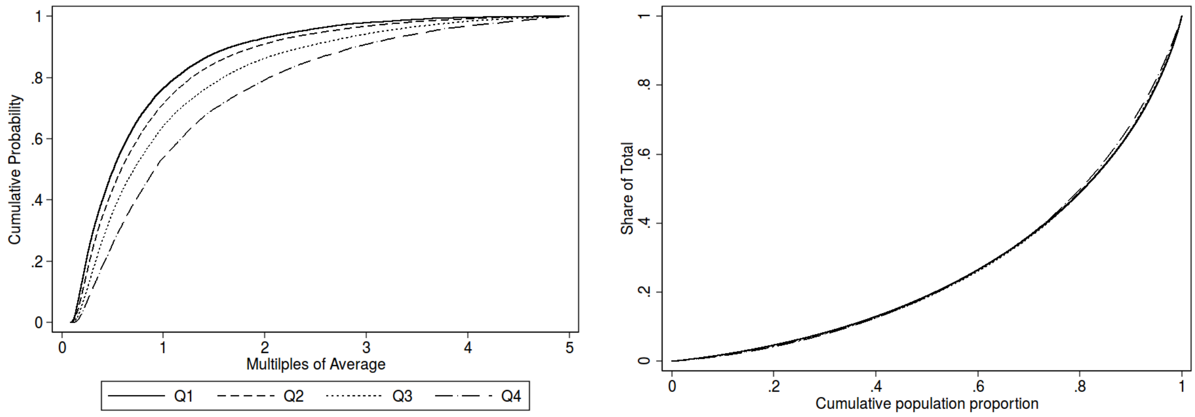
(b) Case 2: conditional distributions of z_h by a_{-1} quartiles



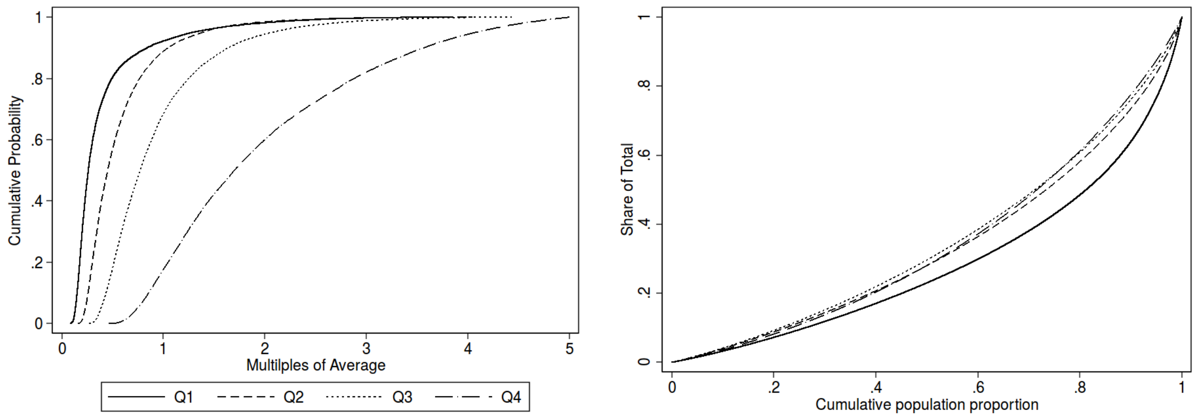
(c) Case 3: conditional distributions of z_w by $z_{h,-1}$ quartiles

Figure F5: Outcome distributions and Lorenz curves by type, cases 1-3

For each case, the variables are demeaned by the unconditional mean.



(a) Case 4: conditional distributions of c_v by a_{-1} quartiles



(b) Case 5: conditional distributions of c_v by $z_{h,-1}$ quartiles

Figure F6: Outcome distributions and Lorenz curves by type, cases 4-5
For each case, the variables are demeaned by the unconditional mean.

than children's income, is qualitatively the same. It also remains true that the distributions of children's income and continuation values conditional on parents' abilities display similar patterns, while children's net wealth and children's continuation values conditional on parents' abilities display similar patterns. Hence the particular employment of concepts to our model are not sensitive to the assumption that luck is correlated to own abilities.

However, it is clear that overall inequality is larger without the negative correlation, as expected from Section 2.2. In particular, note that the relationship between cases 2 and 4 are not as tight as in the benchmark: without market luck compensating for own abilities (i.e., a negative ρ_ϵ), children's lifetime income becomes a poorer proxy for continuation values when conditioning on parents' abilities. Nonetheless, it remains true that the Theil index is

		Theil	MLD
	TOTAL	**4.97**	**0.64**
	Between	49%	49%
	Within	51%	51%
Case 1. $(z_{h,-1}, z_h)$	Q1	2.98	0.33
	Q2	2.48	0.27
	Q3	2.46	0.28
	Q4	3.12	0.42
	Between	13%	19%
	Within	87%	81%
Case 2. (a_{-1}, z_h)	Q1	5.26	0.57
	Q2	4.77	0.53
	Q3	4.94	0.57
	Q4	4.69	0.56
	TOTAL	**3.15**	**0.42**
	Between	53%	49%
	Within	47%	51%
Case 3. $(z_{h,-1}, z_w)$	Q1	3.37	0.32
	Q2	1.81	0.19
	Q3	1.43	0.16
	Q4	1.46	0.18

		Theil	MLD
	TOTAL	**3.15**	**0.38**
	Between	5%	0%
	Within	95%	100%
Case 4. (a_{-1}, c_v)	Q1	3.44	0.36
	Q2	3.41	0.36
	Q3	3.45	0.37
	Q4	3.30	0.37
	Between	53%	51%
	Within	47%	49%
Case 5. $(z_{h,-1}, c_v)$	Q1	3.37	0.29
	Q2	1.81	0.17
	Q3	1.43	0.14
	Q4	1.46	0.15

Table F10: Theil Index/MLD when $\rho_\epsilon = 0$
Theil indices are normalized to lie between 0 and 100. The columns "Between" and "Within" are the percentage contributions of the between- and within-type Theil indices (MLDs) to the total Theil index (MLD). Case $I.(x, y)$ refers to the each case I where x is the type variable and y is the outcome variable.

less sensitive to extreme data points, so that the difference between cases 2 and 4 are less than when using the MLD.

F.2 Policy changes in partial equilibrium

In Table F11, we repeat the policy exercise from Section 4.3, but instead of the computing the long-run, stationary equilibrium outcome of the policy change, we simulate the short-run, one-generation ahead outcome.

As shown in the table, the qualitative effects are similar to the long-run equilibria. That is, i) only subsidizing education improves both mobility and equality, ii) shifting education subsidies to increase lump-sum subsidies decreases average income and wealth, and iii) the quantitative effects of increasing education subsidies are larger. However, the short-run

	B	E	G
IGE	0.594	0.589	0.594
Gini z_h	0.481	0.473	0.481
RW Gini	0.624	0.628	0.624
$\Delta \log \bar{z_h}$	-	0.8%	-0.1%
$\Delta \log \bar{z_w}$	-	0.6%	-0.1%
$\Delta \log \bar{c_v}$	-	0.5%	0.0 %

Table F11: Counterfactual moments (IGE and Gini's) and Average Outcomes
IGE is the correlation of lifetime earnings across generations. z_h is lifetime earnings, and (z_w, c_v) are defined in the text. RW stands for "Retirement Wealth." For each y, $\bar{y}$ refers to its cross-sectional average. B refers to the benchmark stationary equilibrium, and E and G refer to the one-generation ahead outcome: only education subsidies without lump-sum subsidies, and only lump-sum subsidies without education subsidies.

effects are negligible, even for education subsidies.

This is because the one-generation ahead effect is such that parents make their decisions without knowledge of the policy changes.[32] It is only as successive generations update their decision rules knowing that the new policy is in place, that the effect of the policy accumulates resulting in the larger effect seen in the text. Of course in the long-run, this also alters the distribution of human capital and wealth.

[32]This is realistic. Most parents would not alter their childrearing outcomes in response to short-run policy changes.